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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B.Tech S4 (S,FE) / S2 (PT) (S,FE) Examination May 2023 (2015 Scheme)

Course Code: MA202

Course Name: PROBABILITY DISTRIBUTIONS, TRANSFORMS AND NUMERICAL METHODS

Max. Marks: 100 Duration: 3 Hours

Normal distribution table is allowed in the examination hall.

PART A (MODULES I AND II)

Answer two full questions.

1 a) The probability mass function of a random variable X is given below: (7)

X:	-2	-1	0	1	2	3
$f(x)$:	0.1	k	0.2	$2k$	0.3	k

Find (i) value of k (ii) $P(X \geq -1)$ (iii) $E(X)$ (iv) $P(0 < X \leq 3)$

(v) Distribution function of X.

b) The probability that a batsman scores a century in a cricket match is $\frac{1}{3}$. Find the probability that out of 5 matches, he may score century in (8)

(i) at least 2 matches (ii) at most 2 matches (iii) no match.

2 a) A random variable follows Poisson distribution such that $P(X = 0) = \frac{2}{3}P(X = 1)$. (7)

Find (i) $P(X = 3)$ (ii) $P(X > 3)$.

b) Find the value of k and hence find the mean and variance of the distribution (8)

$$f(x) = \begin{cases} kx^3, & 0 < x < 1 \\ 0, & otherwise \end{cases}$$

3 a) Assume that the time between arrivals of customers at a particular bank is exponentially distributed with a mean of 4 minutes (7)

(i) Find the probability that the time between arrivals is greater than 5 minutes.

(ii) Find the probability that the time between arrivals is between 1 and 4 minutes.

b) The marks obtained by a batch of students in a certain subject are normally distributed. 10% of students got less than 45 marks while 5% got more than 75. Find the mean and standard deviation of the distribution. (8)

PART B (MODULES III AND IV)

Answer two full questions.

4 a) Find the Fourier cosine transform of (7)

$f(x) = \begin{cases} 1 - x^2, & |x| \leq 1 \\ 0, & otherwise \end{cases}$. Hence prove that f(x) $= \frac{4}{\pi}\int_0^\infty \left(\frac{\sin\omega - \omega\cos\omega}{\omega^3}\right) \cos\omega x \, d\omega$.

b) Find the Fourier integral representation of the function $f(x) = \begin{cases} 2, & |x| < 2 \\ 0, & |x| > 2 \end{cases}$ and (8)

hence evaluate $\int_0^\infty \left(\frac{\sin 2\omega}{\omega}\right) \cos\omega x d\omega$.

5 a) Using Fourier sine integral for $f(x) = e^{-kx}$, x> $0, k > 0$, (7)

show that $\int_0^\infty \left(\frac{\omega \sin\omega x}{k^2+\omega^2}\right) d\omega = \frac{\pi}{2} e^{-kx}$.

b) Find the Laplace transform of (8)

(i) $3e^{5t} + (t+2)^2 +$ 2cos3t (ii) $4te^{-2t}$

6 a) Using convolution theorem find the inverse Laplace transform of $\frac{s^2}{(s^2+a^2)(s^2+b^2)}$. (7)

b) Solve the initial value problem, using Laplace transforms (8)

$y'' - 2y' + 2y = 0$ where $y(0) = y'(0) = 1$.

PART C (MODULES V AND VI)

Answer two full questions.

7 a) Find the value of $\sqrt[3]{24}$ using Newton Raphson Method. (6)

b) Using Lagrange's formula for interpolation find the value of y when $x = 2$ from the following table (7)

x : -2 -1 0 4

y : -2 4 1 8

c) Using Newton's Interpolation formula find the value of tan (0.26) from the following data (7)

x	0.10	0.15	0.20	0.25	0.30
$tanx$	0.1003	0.1511	0.2027	0.2553	0.3093

8 a) Using Euler's method to find $y(0.2)$ and $y(0.4)$,given $y' = x + y, y(0) = 1, h = 0.2$ (6)

b) Solve the following by Gauss-Seidel Method (7)

$10x+y+z = 12$

$2x+10y+z = 13$

$2x+2y+10z = 14$

c) Evaluate $\int_0^2 e^{-x}dx$ by using Simpson's rule with 4 subintervals and compare it with the exact solution. (7)

9 a) Find the unique polynomial $p_3(x)$ of degree 3 or less, the graph of which passes through the points (-1,3), (0,-4), (1,5) and (2,-6). (6)

Solve the equations using Gauss elimination method

b) $x+2y+z=3$ (7)

$2x+3y+2z=5$

$3x-5y+5z=2$

$3x+9y-z=4$

c) Obtain the value of y at $x = 0.2$ using Runge- Kutta method of fourth order for the differential equation $\frac{dy}{dx} = 1 + y^2$ with h = 0.2, y(0) = 0 . (7)

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