Reg No.:_______________ Name:_______________________

221TCS002022301

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

First Semester M.Tech Degree Examination December 2022 (2022 Scheme)

Discipline: COMPUTER SCIENCE AND ENGINEERING

Course Code & Name: 221TCS002 FOUNDATIONS OF COMPUTER SCIENCE

Normal distribution tables are permitted during the examination

Max. Marks: 60 Duration: 2.5 Hours

PART A

Answer all questions. Each question carries 5 marks Marks

1 Use mathematical induction to show that $2n + 1 < 2^n$ for all integers $n > 3$. (5)

2 One hundred tickets numbered 1, 2, 3, ... , 100 are sold to 100 different people. Four different prizes, including a grand prize are awarded after a draw. How many ways are there to award the prizes if (5)

- (i) there are no restrictions?
- (ii) the person holding ticket 47 wins the grand prize?
- (iii) the person holding ticket 47 wins one of the prizes?
- (iv) the person holding ticket 47 does not win a prize?
- (v) the people holding tickets 19 and 47 both win prizes?

3 A salad can be made using at most four oranges, at most one pears, any even number of apples and bananas in groups of five. Using generating functions, find the number of ways can we make a salad with n of these fruits subject to the listed constraints. (5)

4 The number of accidents in a national highway per week is known to follow a Poisson distribution with mean 0.5. Find the probability that (5)

- (i) in a particular week there will be less than 2 accidents.
- (ii) in a particular week there will be more than 2 accidents.
- (iii) in a three week period there will be no accidents.

5 Find the probability that among n people there are at least two people with the same birthday in a leap year. (5)

PART B

Answer any 5 questions. Each question carries 7 marks

6 (a) A bowl contains 10 red balls and 10 blue balls. A woman selects balls at random without looking at them.

(i) How many balls must she select to be sure of having at least three balls of the same color? (4)

(ii) How many balls must she select to be sure of having at least three blue balls?

(b) Show that if n is even, then $n + 4$ is even using (3)

(i) a direct proof (ii) a proof by contraposition (iii) a proof by contradiction.

7 Using principle of inclusion and exclusion, find the number of solutions of the equation $x_1 + x_2 + x_3 = 10$ where x_1, x_2 and x_3 are nonnegative integers with $x_1 \leq 3$, $x_2 \leq 4$, and $x_3 \leq 6$. (7)

8 (a) Is the set of rational numbers countable? Prove your answer. (3)

(b) Are all infinite sets countable? Justify your answer with an example and prove the validity of your example. (4)

9 Using generating functions, solve the following recurrence relation: (7)

$a_n = 5a_{n-1} - 6a_{n-2}, n \geq 2$, given that $a_0 = 1$ and $a_1 = -2$.

10 (a) A factory produces nails with mean 4 and standard deviation 0.3, assuming normal distribution, where measurements are in mm. Nails smaller than 3.5 mm or bigger than 4.4 mm are rejected. Out of a batch of 500 nails, how many would be acceptable? (4)

(b) The random variable X is uniformly distributed over $(-2, 2)$. Find (3)

(i) $P(|X| > 1)$ (ii) $P(2X + 3 > 5)$

11 (a) A light bulb manufacturing factory finds 3 in every 60 light bulbs defective. What will be the probability that the first defective light bulb will be found when the 6th one is tested? (3)

(b) State and prove Lagrange's Theorem. (4)

12 There are n distinct coupons placed in an urn. Coupons are randomly selected one at a time (with replacement) until at least one of each type of coupon has been selected. Find the expected number of selections made until all distinct n coupons are collected. (7)

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