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# APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B.Tech S1 (S,FE) S2 (S) Examination February 2023 (2015 Scheme)

# Course Code: MA102 Course Name: DIFFERENTIAL EQUATIONS

| Max. Marks: 100 |                                                                                                                    | Duration: 3 Hours |  |  |
|-----------------|--------------------------------------------------------------------------------------------------------------------|-------------------|--|--|
|                 | PART A                                                                                                             |                   |  |  |
| 1               | Answer all questions, each carries 3 marks<br>Find the general solution of the differential equation $y'' + y = 0$ | (3)               |  |  |
| 2               | Reduce to first order and solve $yy'' = 3(y')^2$ .                                                                 | (3)               |  |  |
| 3               | Find the particular solution of the differential equation $y''' + y = cos2x$ .                                     | (3)               |  |  |
| 4               | Using a suitable transformation, convert the differential equation                                                 |                   |  |  |
|                 | $(x+1)^2y'' + (x+1)y' + y = 4\cos(\log(1+x))$ into a linear differential                                           | (3)               |  |  |
|                 | equation with constant coefficients.                                                                               |                   |  |  |
| 5               | Find the Fourier series of the function $f(x) = x + \pi$ in the range $-\pi < x < \pi$ .                           | (3)               |  |  |
| 6               | Find the Fourier sine series of $f(x) = x$ , in $0 \le x \le 3$ .                                                  | (3)               |  |  |
| 7               | Find the partial differential equation of all spheres having their centre lies on z-                               | (2)               |  |  |
|                 | axis.                                                                                                              | (3)               |  |  |
| 8               | Solve $2r + 5s + 2t = 0$ .                                                                                         | (3)               |  |  |
| 9               | Solve $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$ using method of separation of variables. | (3)               |  |  |
| 10              | Write the three possible solutions of one dimensional wave equation.                                               | (3)               |  |  |
| 11              | Find the steady state temperature distribution in a rod of 50 cm having its ends at                                | (3)               |  |  |
|                 | 20°C and 70°C respectively.                                                                                        | (J)               |  |  |
| 12              | Write the three basic laws which are used in the derivation of one dimensional                                     | (3)               |  |  |
|                 | heat equation.                                                                                                     |                   |  |  |
| PART B          |                                                                                                                    |                   |  |  |

# Answer six questions, one full question from each module

### Module 1

13 a) Solve the initial value problem

$$y''' + y'' + 4y' + 4y = 0, y(0) = 2, y'(0) = -5, y''(0) = -3.$$
(5)

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|            | b)       | Find a basis of solutions of the differential equation $y'' - 3y' + 2y = 0$                                          | (6)  |  |  |
|------------|----------|----------------------------------------------------------------------------------------------------------------------|------|--|--|
| OR         |          |                                                                                                                      |      |  |  |
| 14         | a)       | Find a basis of solution of the ODE $(x^2)y'' + xy' - 4y = 0$ if $y_1 = x^2$ is a                                    |      |  |  |
|            |          | solution.                                                                                                            | (5)  |  |  |
|            | b)       | Solve $y^{\nu} - 3y'^{\nu} + 3y''' - y'' = 0$ .                                                                      | (6)  |  |  |
| Module 1I  |          |                                                                                                                      |      |  |  |
| 15         | a)       | Solve $x^2y'' - xy' + 4y = \cos(\log x) + x\sin(\log x)$ .                                                           | (5)  |  |  |
|            | b)       | Use method of variation of parameters solve $y'' + y = secx$ .                                                       | (6)  |  |  |
| OR         |          |                                                                                                                      |      |  |  |
| 16         | a)       | Solve $y'' - 4y' + 4y = 8x^2 e^{2x} sin 2x$ .                                                                        | (5)  |  |  |
|            | b)       | Solve $y'' + 4y = tan2x$ .                                                                                           | (6)  |  |  |
| Module 1II |          |                                                                                                                      |      |  |  |
| 17         |          | Find the Fourier series for the function $f(x) = \begin{cases} x, & 0 < x < 1 \\ 1-x, & 1 < x < 2 \end{cases}$ Hence | (11) |  |  |
|            |          | deduce that $1 + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$ .                                          | (11) |  |  |
| OR         |          |                                                                                                                      |      |  |  |
| 18         | a)       | Find the Fourier series of $f(x) = \begin{cases} 1-x, -\pi < x \le 0\\ 1+x, 0 \le x \le \pi \end{cases}$ .           | (6)  |  |  |
|            | b)       | Find the Fourier cosine series of $f(x) = cosx \text{ in } 0 < x < \frac{\pi}{2}$ .                                  | (5)  |  |  |
| Module 1V  |          |                                                                                                                      |      |  |  |
| 19         | a)       | Solve $(y^2(x+y))p + (x^2(x+y))q = (x^2 + y^2)z$ .                                                                   | (5)  |  |  |
|            | b)       | Find the particular integral of $(D^3 - 10D^2D' + D'^3) z = \cos(2x+3y)$                                             | (6)  |  |  |
| ٠          |          | OR                                                                                                                   |      |  |  |
| 20         | a)       | Solve $xy dx + y^2 dy = zxy - 2x^2$                                                                                  | (5)  |  |  |
|            | b)       | Solve $(D^2 + 3DD' + 2{D'}^2)z = x^2y^2$                                                                             | (6)  |  |  |
|            | Module V |                                                                                                                      |      |  |  |
| 21         |          | Derive one dimensional wave equation.                                                                                | (10) |  |  |
|            | 2        | OR                                                                                                                   |      |  |  |
| 22         |          | Find the deflection of the vibrating string which is fixed at the ends $x=0$ and $x=2$                               | (10) |  |  |
|            |          |                                                                                                                      |      |  |  |

and the motion is started by displacing the string into the form  $sin^3 \frac{\pi x}{2}$  and released it with zero initial velocity at t=0.

#### **Module VI**

The ends A and B of a rod of length 20 cm are maintained at temperatures  $30^{\circ}$ C and  $80^{\circ}$ C respectively until steady state conditions prevail. The temperature of the ends are changed to  $40^{\circ}$ C and  $60^{\circ}$ C respectively. Find the temperature distribution in the rod at time t.

### (10)

#### OR

Find the temperature distribution in a bar of length  $\pi$  whose surface is thermally insulated with end points maintained at 0°C. The initial temperature distribution in

the rod is 
$$u(x,0) = \begin{cases} x, 0 \le x \le \frac{\pi}{2} \\ \pi - x, \frac{\pi}{2} \le x \le \pi \end{cases}$$

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### (10)