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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Third Semester B.Tech Degree Regular and Supplementary Examination December 2022 (2019 scheme)

Course Code: MAT203

Course Name: Discrete Mathematical Structures

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions. Each question carries 3 marks Marks

1 Using truth table, verify the logical equivalence (3)

$$[p \rightarrow (qvr] \Leftrightarrow [\neg r \rightarrow (p \rightarrow q)]$$

2 Let $p(x)$ and $q(x)$ denote the open statements in the universe of all integers (3)

$$p(x): x^2 - 2x - 3 = 0 \quad \& \quad q(x): x < 0$$

Determine the truth value of the following statement. If the statement is false provide a suitable counterexample,

$$\forall x[p(x) \rightarrow q(x)]$$

3 How many arrangements are there of all the letters in 'SOCIOLOGICAL' such that A and G are adjacent? (3)

4 A machine that inserts letters into envelopes goes haywire and inserts letters randomly into envelopes. What is the probability that in a group of 100 letters, no letter is put in the correct envelope? (3)

5 Determine whether the function $f: Z \rightarrow Z, f(x) = 2x$ is one-to-one and find its range. (3)

6 Define partial ordering relation. Give an example. (3)

7 Write a recurrence relation for the number of words of length *n*, using only the three letters *a, b, c* which does not have two consecutive *a*'s. (3)

8 Find the exponential generating function for the sequence 1,-1, 1,-1, 1,-1, ... (3)

9 Let $S = \{0,1,2,3\ 4 \ldots\}$. Is $(S, +)$ a monoid? (3)

10 State Lagrange's Theorem. (3)

PART B

Answer any one full question from each module. Each question carries 14 marks

Module 1

11(a) Without using truth tables, show that $(p \vee q) \wedge \neg(\neg p \wedge q) \Leftrightarrow p$ (6)

(b) Prove the validity of the following argument: (8)

"If it does not rain or if there is no traffic dislocation, then the sports day will be held and cultural programmes will go on"; "If the sports day is held, the trophy will be awarded" and "the trophy was not awarded"

Therefore "It rained".

12(a) Show that the following argument is invalid (6)

$$p\ ,\ p \vee q\ ,\ q \longrightarrow (r \longrightarrow s)\ ,\ t \longrightarrow r \text{ Therefore } \neg s \longrightarrow \neg t$$

(b) Determine the truth value of each of the following statements if the universe comprises all non zero integers (8)

(i)$\exists x \exists y[xy = 2]$

(ii) $\exists x \forall y[xy = 2]$

(iii) $\exists x \exists y[(4x + 2y = 3) \wedge (x - y = 1)]$

Module 2

13(a) Let $S \subset Z^+$, where $|S| = 37$. Show that S contains two elements that have the same remainder upon division by 36 (6)

(b) Determine the number of integer solutions of $x_1 + x_2 + x_3 + x_4 = 32$, such that (i) $x_i \geq 0,\ 1 \leq i \leq 4$ (8)

(ii) $x_i > 0,\ 1 \leq i \leq 4$

(iii) $x_i \geq -2,\ 1 \leq i \leq 4$

14(a) In the expansion of $(x + y + z)^7$, determine the coefficient of (6)

(i) $x^2y^2z^2$

(ii) x^3z^4

(b) During a 12 week conference, the vice-chancellor (V.C.) met his seven friends from college. During the conference, V.C. met each friend at lunch 35 times, every pair of them 16 times, every trio 8 times, every foursome 4 times, each set of five twice, each set of six once, but never all seven at once. If he had lunch every day during the 84 days of conference, did he ever have lunch alone. (8)

Module 3

15(a) Let $A = \{1,2,3,4,6,8,12\}$ Let R be a partial ordering on A defined by $a\,R\,b$ if a divides b. Draw a Hasse diagram for the poset *(A, R)* (6)

(b) If $A = \{1,2,3,4,5,6,7\}$ define $\mathcal{R}$ on A by $(x, y) \in \mathcal{R}$ if $x - y$ is a multiple of 3. Show that R is an equivalence relation on A (8)

16(a) Let $A = \{a, b, c\}$. (6)

(i) List the set of all proper subsets of A.

(ii) Let $\mathcal{R}$ be the subset relation defined on the set of all proper subsets of A. Find the maximal elements and minimal elements

(b) Let $A = \{1,2,3,12\}$. Show that A under division is a lattice. (8)

Module 4

17(a) Solve the recurrence relation $a_{n+2} = 4a_{n+1} - 4a_n \quad n \geq 0 \quad a_0 = 1, a_1 = 3$ (6)

(b) Solve the recurrence relation (8)

$$a_{n+2} - 3a_{n+1} + 2a_n = 2^n \quad n \geq 0 \quad a_0 = 3, a_1 = 6$$

18(a) In how many ways can a police officer distribute 24 rifle shells to four police officers so that each officer gets at least three shells but not more than eight? (6)

(b) Solve the recurrence relation $a_{n+1} = a_n + n, \quad n \geq 2, \quad a_2 = 1$ (8)

Module 5

19(a) Show that any group G is abelian if and only if $(ab)^2 = a^2b^2$ for all $a, b \in G$ (6)

(b) Let $G = \{1, -1, i, -i\}$ and . defines multiplication (8)

(i) Show that $(G, .)$ a group .

(ii) Is $(G, .)$ a cyclic group. If so, find each generator of G.

(iii) Find the order of each element in G.

20(a) Define semigroup homomorphism. Determine which of the following functions are homomorphism from $(Z^+, +)$ to $(Z^+, \cdot)$ (6)

(i) $f(n)=n$

(ii) $f(n)=3^n$

(b) Prove that " If H is a non-empty subset of a group G then H is a subgroup of G if and only if (i) for all $a, b \in H, ab \in H$ and (ii) for all $a \in H, a^{-1} \in H$." (8)

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