Reg No.:_______________ Name:_______________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Third Semester B.Tech Degree Regular and Supplementary Examination December 2022 (2019 Scheme)

Course Code: MAT201

Course Name: Partial Differential equations and Complex analysis

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions. Each question carries 3 marks Marks

1	Form a partial differential equation from the relation $z = (x+y)f(x^2 - y^2)$.	(3)
2	Solve $\frac{\partial^3 z}{\partial x^2 \partial y} = \cos(3x + 4y)$.	(3)
3	Write any three assumptions involved in the derivation of one dimensional wave equation.	(3)
4	Find the steady state temperature distribution in a rod of length 25 cm, if the ends of the rod are kept at $20°c$ and $70°c$.	(3)
5	Determine whether $w = \cos z$ is analytic.	(3)
6	Check whether the function xy^2 is the real part of an analytic function.	(3)
7	Using Cauchy's integral formula, Evaluate $\int_C \frac{z^2+1}{z^2-1}\, dz$ where C is the circle of unit radius with centre at $z = 1$.	(3)
8	Find the Taylor's series of $\frac{1}{z}$ about the point $z = 1$.	(3)
9	Find the Laurent series of $z^2 e^{1/z}$ about $z = 0$ and determine the region of convergence.	(3)
10	Find the zeros and their order of the function $sin^2(z)$.	(3)

PART B

Answer any one full question from each module. Each question carries 14 marks

Module 1

11	(a)	Find the differential equation of all planes which are at a constant distance 'c' from the origin.	(7)
	(b)	Solve $y^2 p - xyq = x(z - 2y)$	(7)

12 (a) Solve $pq + 2x(y+1)p + y(y+2)q - 2(y+1)z = 0$. (7)

(b) Solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ where $u(x,0) = 3e^{5x}$ by method of separation of variables. (7)

Module 2

13 (a) A tightly stretched string of length one cm is fastened at both ends. Find the displacement of a string if it is released from rest from the position $\sin \pi x + 5\sin 3\pi x$. (7)

(b) A rod of $30\ cm$ long has its ends A and B kept at $30°c$ and $90°c$ respectively until steady state temperature prevails. The temperature at each end is then suddenly reduced to zero temperature and kept so. Find the resulting temperature function $u(x,y)$, taking $x = 0$ at A. (7)

14 (a) A tightly stretched homogeneous string of unit length with its fixed ends at $x = 0\ and\ x = 1$ executes transverse vibrations. The initial velocity is zero and the initial deflection is given by $u(x,0) = \begin{cases} 1, & 0 \le x < \frac{1}{2} \\ -1, & \frac{1}{2} \le x \le 1 \end{cases}$. Find the deflection $u(x,t)$ at any time t. (7)

(b) Derive one dimensional heat equation (7)

Module 3

15 (a) Find the image of the semi circle $y = \sqrt{4 - x^2}$ under the transformation $w = z^2$. (7)

(b) Show that $u = x^2 - y^2 - y$ is harmonic. Also find the corresponding harmonic conjugate function. (7)

16 (a) Find the image of the circle $|z - 1| = 1$ under the mapping $w = \frac{1}{z}$. (7)

(b) If $f(z) = u(x,y) + iv(x,y)$ ~~is~~ analytic and $uv = 2023$, then show that $f(z)$ is a constant (7)

Module 4

17 (a) Using Cauchy's integral formula, Evaluate the integral $\int_C \frac{2z+3}{z^2} dz$, where C is a circle $|z - i| = 2$ counter clockwise. (7)

(b) Evaluate $\int_C (z^2 + 3z)\, dz$ along the circle $|z| = 2$ from (2,0) to (0,2) in counter-clockwise direction. (7)

18	(a)	Using Cauchy's integral formula, Evaluate $\oint_C \frac{e^{2z}}{(z+1)^4}\,dz$ where C is the circle $\lvert z\rvert = 2$.	(7)
	(b)	Expand $f(z) = \frac{z+1}{z-1}$ as a Taylor series about $z = -1$	(7)

Module 5

19	(a)	Find the Laurent series expansion of $f(z) = \frac{1}{1-z^2}$ about $z = 1$ in the regions (i) $0 < \lvert z-1\rvert < 2$ (ii) $\lvert z-1\rvert > 2$	(7)
	(b)	Evaluate $\int_0^{2\pi} \frac{1}{2+\cos\theta}\,d\theta$.	(7)
20	(a)	Using Cauchy's Residue theorem, Evaluate $\int_C \frac{30z^2-23z+5}{(2z-1)^2(3z-1)}\,dz$ where C is the circle $\lvert z\rvert = 1$ counter-clockwise.	(7)
	(b)	Using contour integration, evaluate $\int_{-\infty}^{\infty} \frac{1}{(1+x^2)^3}\,dx$.	(7)

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