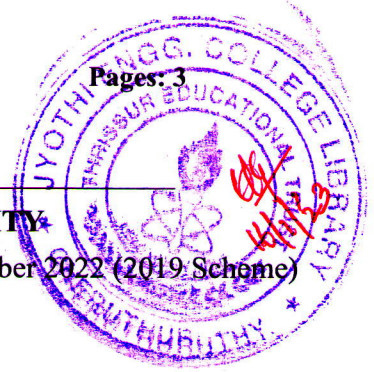


Reg No.: _____

Name: _____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Third Semester B.Tech Degree Regular and Supplementary Examination December 2022 (2019 Scheme)



Course Code: MAT201

Course Name: Partial Differential equations and Complex analysis

Max. Marks: 100

Duration: 3 Hours

PART A

Answer all questions. Each question carries 3 marks

Marks

- 1 Form a partial differential equation from the relation $z = (x + y)f(x^2 - y^2)$. (3)
- 2 Solve $\frac{\partial^3 z}{\partial x^2 \partial y} = \cos(3x + 4y)$. (3)
- 3 Write any three assumptions involved in the derivation of one dimensional wave equation. (3)
- 4 Find the steady state temperature distribution in a rod of length 25 cm, if the ends of the rod are kept at 20°C and 70°C . (3)
- 5 Determine whether $w = \cos z$ is analytic. (3)
- 6 Check whether the function xy^2 is the real part of an analytic function. (3)
- 7 Using Cauchy's integral formula, Evaluate $\int_C \frac{z^2+1}{z^2-1} dz$ where C is the circle of unit radius with centre at $z = 1$. (3)
- 8 Find the Taylor's series of $\frac{1}{z}$ about the point $z = 1$. (3)
- 9 Find the Laurent series of $z^2 e^{1/z}$ about $z = 0$ and determine the region of convergence. (3)
- 10 Find the zeros and their order of the function $\sin^2(z)$. (3)

PART B

Answer any one full question from each module. Each question carries 14 marks

Module 1

- 11 (a) Find the differential equation of all planes which are at a constant distance 'c' from the origin. (7)
- (b) Solve $y^2 p - xyq = x(z - 2y)$ (7)

- 12 (a) Solve $pq + 2x(y + 1)p + y(y + 2)q - 2(y + 1)z = 0$. (7)
- (b) Solve $\frac{\partial u}{\partial x} = 2 \frac{\partial u}{\partial t} + u$ where $u(x, 0) = 3e^{5x}$ by method of separation of variables. (7)

Module 2

- 13 (a) A tightly stretched string of length one cm is fastened at both ends. Find the displacement of a string if it is released from rest from the position $\sin \pi x + 5 \sin 3\pi x$. (7)
- (b) A rod of 30 cm long has its ends A and B kept at 30°C and 90°C respectively until steady state temperature prevails. The temperature at each end is then suddenly reduced to zero temperature and kept so. Find the resulting temperature function $u(x, y)$, taking $x = 0$ at A. (7)
- 14 (a) A tightly stretched homogeneous string of unit length with its fixed ends at $x = 0$ and $x = 1$ executes transverse vibrations. The initial velocity is zero and the initial deflection is given by $u(x, 0) = \begin{cases} 1, & 0 \leq x < \frac{1}{2} \\ -1, & \frac{1}{2} \leq x \leq 1 \end{cases}$. Find the deflection $u(x, t)$ at any time t . (7)
- (b) Derive one dimensional heat equation (7)

Module 3

- 15 (a) Find the image of the semi circle $y = \sqrt{4 - x^2}$ under the transformation $w = z^2$. (7)
- (b) Show that $u = x^2 - y^2 - y$ is harmonic. Also find the corresponding harmonic conjugate function. (7)
- 16 (a) Find the image of the circle $|z - 1| = 1$ under the mapping $w = \frac{1}{z}$. (7)
- (b) If $f(z) = u(x, y) + iv(x, y)$ is analytic and $uv = 2023$, then show that $f(z)$ is a constant (7)

Module 4

- 17 (a) Using Cauchy's integral formula, Evaluate the integral $\int_C \frac{2z+3}{z^2} dz$, where C is a circle $|z - i| = 2$ counter clockwise. (7)
- (b) Evaluate $\int_C (z^2 + 3z) dz$ along the circle $|z| = 2$ from $(2, 0)$ to $(0, 2)$ in counter-clockwise direction. (7)

18 (a) Using Cauchy's integral formula, Evaluate $\oint_C \frac{e^{2z}}{(z+1)^4} dz$ where C is the circle $|z| = 2$. (7)

(b) Expand $f(z) = \frac{z+1}{z-1}$ as a Taylor series about $z = -1$. (7)

Module 5

19 (a) Find the Laurent series expansion of $f(z) = \frac{1}{1-z^2}$ about $z = 1$ in the regions (7)

(i) $0 < |z - 1| < 2$ (ii) $|z - 1| > 2$

(b) Evaluate $\int_0^{2\pi} \frac{1}{2+\cos\theta} d\theta$. (7)

20 (a) Using Cauchy's Residue theorem, Evaluate $\int_C \frac{30z^2-23z+5}{(2z-1)^2(3z-1)} dz$ where C is the circle $|z| = 1$ counter-clockwise. (7)

(b) Using contour integration, evaluate $\int_{-\infty}^{\infty} \frac{1}{(1+x^2)^3} dx$. (7)
