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Reg No.:_____ Name:_____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Fourth Semester B.Tech Degree Examination June 2022 (2019 scheme)

Course Code: MAT256

Course Name: PROBABILITY AND STATISTICAL MODELLING

Max. Marks: 100 **Duration: 3 Hours**

(Statistical tables are allowed)

PART A

(Answer all questions; each question carries 3 marks) Marks

1 A gambler plays a game of rolling a die with the following rules. He will win Rs.200 if he throws a 6, but will lose Rs.40 if he throws 4 or 5 and lose Rs.20 if he throws 1,2, or 3. Find the expected value that the gambler may gain. 3

2 If the mean of a binomial distribution for 5 trials is 0.8, find the probability distribution. 3

3 Find the value of k for the probability density function f(x) given below and hence find its mean and variance where 3

$$f(x)=\begin{cases} kx^3 & 0 < x < 1 \\ 0 & otherwise \end{cases}$$

4 If random variable X has a uniform distribution in(-3,3), find $p(|X-2|<2)$ 3

5 A sample of 16 measurements of the diameter of a sphere gave a mean $\bar{X}$=4.58 inches and a standard deviation σ_s=0.08 inches. Find a 95% confidence limits for the actual diameter. 3

6 Discuss the difference between F distribution and students t-distribution 3

7 Define (i) type I and type II errors, (II) level of significance. 3

8 Define (i) 'P-value' (ii) Power of the test 3

9 In the following data, x=current density (Ma/cm^2) and y=rate of deposition (μm/min), Calculate coefficient of correlation. 3

X	20	40	60	80
Y	0.24	1.20	1.71	2.22

10 Calculate the regression coefficients and obtain the lines of regression for the following data 3

X	1	2	3	4	5
Y	2	5	11	8	14

PART B

(Answer one full question from each module, each question carries 14 marks)

Module -1

11 a) Find the Mean and Variance of Binomial Distribution 7

b) It is known that 2% of the accounts in a company delinquent. If 5 accounts are selected at random, compute the following probabilities (i) at most 2 accounts will be delinquent (ii) at most 4 accounts will be delinquent. 7

12 a) A discrete random variable X has the mean 6 and variance 2. If it is assumed that the distribution is binomial find 7

$P[5 \leq X \leq 7]$ (ii)$P[X \leq 2]$ (iii)$P[X > 7]$

b) The joint distribution of (X, Y) is given by $f(x,y) = \frac{x+y}{21}$, x=1, 2, 3 and y= 1, 2. Then find the marginal distributions. Also find E(X) and E(Y). 7

Module -2

13 a) The amount of time that a surveillance camera will run without having to be reset is a random variable having the exponential distribution with mean 50 days. Find the probability that such camera will 7

(i) Have to be reset less than 20 days

(ii) Not have to be reset in at least 60 days

(iii) Have to be reset between 20 and 60 days

b) Random variable has normal distribution with $\mu = 62.4$. Find it's standard deviation if the probability is 0.2 that it will take on a value greater than 79.2 7

14 a) 5% of the observation in normal distribution are below 5 and 25% of the observations are below 25. Find mean and SD 7

b) The life of a certain brand of tube light may be considered as random variable with mean 1200 hours and standard deviation 250 hours. Using Central limit theorem find the probability that the average life time of 60 lights exceeds 1250. 7

Module -3

15 a) Determine the size of the sample for estimating the true weight of the cereal containers for the universe with N = 5000 .Given variance of weight = 4 ounces on the basis of past records and estimate should be within 0.8 ounces of the true average weight with 99% probability. Will there be a change in the size of the sample if we assume infinite population in the given case? If so, explain by how much? 7

b) From a random sample of 36 New Delhi civil service personnel, the mean age and the sample standard deviation were found to be 40 years and 4.5 years respectively. Construct a 95 per cent confidence interval for the mean age of civil servants in New Delhi. 7

16 a) A market research survey in which 64 consumers were contacted states that 64 per cent of all consumers of a certain product were motivated by the product's advertising. Find the confidence limits for the proportion of consumers motivated by advertising in the population, given a confidence level equal to 0.95. 7

b) What should be the size of the sample if a simple random sample from a population of 4000 items is to be drawn to estimate the per cent defective within 2 per cent of the true value with 95.5 per cent probability? What would be the size of the sample if the population is assumed to be infinite in the given case? 7

Module -4

17 a) A company claims that the mean thermal efficiency of diesel engines produced by them is 32.3%. To test this claim, random sample of 40 engines were examined which showed the mean thermal efficiency of 31.4% and Standard deviation of 1.6%. Can the claim be accepted or not. Use 0.01 level of significance? 7

b) Mean life time of computers manufactured by a company is 1120 hours with standard deviation of 125 hours. Test the hypothesis that mean lifetime of computers has not changed if a random sample of 8 computers has a mean life time of 1070 hours. Use 0.05 level of significance. 7

18 a) Test the claim of a manufactures that 95% of his 'stabilizers' confirm to ISI specifications, if out of a random sample of 200 stabilizers produced by this manufacturer 18 were faulty. Use 0.05 level of significance. 7

b) The accompanying data resulted from an experiment comparing the degree of soiling for fabric copolymerized with three different mixtures of methacrylic acid. Test whether the mixtures appear to be indistinguishable with respect to degree of soiling. Use 0.01 level of significance. 7

Mixture 1	.56	1.12	.90	1.07	.94
Mixture 2	.72	.69	.87	.78	.91
Mixture 3	.62	1.08	1.07	.99	.93

Module -5

19 a) The flow rate y (m^3/min) in a device used for air-quality measurement depends on the pressure drop x (in. of water)across the device's filter. Suppose that for x values between5 and 20, the two variables are related according to the simple linear regression model with true regression line y= -0.12+0.095x 7

What is the expected change in flow rate associated with1-in. increase and 5 in. decrease in pressure drop? Explain

b) The flow rate y (m^3/min) in a device used for air-quality measurement depends on the pressure drop x (in. of water) across the device's filter. Suppose that for x values between5 and 20, the two variables are related according to the simple linear regression model with true regression line y= -0.12+0.095x. 7

Suppose σ=0.025 and consider a pressure drop of 10 in. What is the probability that the observed value of flow rate will exceed 0.835?. What is the probability that the observed flow rate will exceed 0.840?

20 a) Suppose the variables x= commuting distance and y= commuting time are related according to the simple linear regression model with σ=10. If n=5 observations are made at the x values x_1= 5, x_2=.10, x_3= 15, x_4= 20, and x_5=25 calculate the standard deviations of the five corresponding residuals. 7

b) The following data on mass rate of burning x and flame length y is representative of that which appeared in an article. 7

x	1.7	2.2	2.3	2.6	2.7	3.0	3.2	3.3	4.1	4.3	4.6	5.7	6.1
y	1.3	1.8	1.6	2.0	2.1	2.2	3.0	2.6	4.1	3.7	5.0	5.8	5.3

(i) Estimate the parameters of a power function model.

(ii) Construct diagnostic plots to check whether a power function is an appropriate model choice.

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