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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Fourth Semester B.Tech Degree Examination June 2022 (2019 scheme)

Course Code: MAT204

Course Name: PROBABILITY, RANDOM PROCESSES AND NUMERICAL METHODS

Max. Marks: 100 Duration: 3 Hours

(Statistical Tables are allowed)

PART A

(Answer all questions; each question carries 3 marks) Marks

1 The probabilities that there will be 0, 1, 2, 3 power failures for a certain machine in the month of June are 0.4, 0.3, 0.2, 0.1 respectively. Find the mean and variance for the number of failures. 3

2 If X is a Poisson variable such that $P[X=1]=P[X=2]$, then find $P[X=3]$. 3

3 A continuous random variable X is uniformly distributed in $(-k,k)$. Find k if $P[X \geq 2]=0.25$. 3

4 If $X_1, X_2, \dots, X_n$ are random variables with mean $\mu=2$ and variance $\sigma^2=2$, then use central limit theorem to estimate $P[110 \leq S_n \leq 150]$, where $S_n = X_1+X_2+\dots+X_n$ and $n=75$. 3

5 A random process is defined by $X(t)=A\cos\omega t, t \geq 0$ where ω is a constant and A is uniformly distributed in $(0,3)$. Determine $E[X(t)]$. 3

6 A random process X(t) has the auto correlation function $R_X(\tau)=25+\frac{8}{4+\tau^2}$. Find the mean-square value and variance of the process. 3

7 Write the Newton-Raphson iteration formula to find the cubic root of a positive number N. 3

8 Use trapezoidal rule to evaluate $\int_0^1 y\,dx$ for the following data. 3

x	0	0.2	0.4	0.6	0.8	1
y	0	0.04	0.16	0.36	0.64	1

9 Write the normal equations obtained by the method of least squares for fitting a parabola $y=a+bx+cx^2$. 3

10 Given the initial value problem, $y'=f(x,y)$, with $y(x_0)=y_0$. Write the second order Runge-Kutta algorithm to find the value of y when $x=x_0+h$. 3

PART B

(Answer one full question from each module, each question carries 14 marks)

Module -1

11 a) Derive the mean and variance of binomial distribution. 7

b) The number of gamma rays emitted per second by a certain radioactive substance follows a Poisson distribution with mean 8. Determine the probability that (i) three particles are emitted in one second (ii) at most one particle is emitted in one second (iii) more than one particle is emitted in one second. 7

12 a) A random variable X takes the values -3, -2, -1,0,1,2,3 such that P(X=0) = P(X>0) = P(X<0) and P(X=-3) = P(X=-2) = P(X=-1) = P(X=1) = P(X=2) = P(X=3). Obtain the probability mass function and distribution function of X. 7

b) The joint probability mass function of two random variables X and Y is given by 7

$$p(x,y) = \begin{cases} k(x+2y) & for\ x = 1,2,3\ \ y = 1,2 \\ 0 & otherwise \end{cases}$$ where k is a constant.

(i) Find the value of k (ii) Find $P[X + Y \leq 3]$ (iii) Find the marginal density functions of X and Y and (iv) Are X and Y independent?

Module -2

13 a) A continuous random variable has the distribution function 7

$$F(x) = \begin{cases} 0 & if\ x < 0 \\ k(x-1)^3 & if\ 0 \leq x \leq 4 \\ 1 & if\ x > 4 \end{cases}$$. Find (i) value of k (ii) probability density function $f(x)$ of $F(x)$ and (iii) $P[X \geq 1]$.

b) Suppose the diameter at breast height (in.) of trees of a certain type is normally distributed with mean 8.8 and standard deviation 2.8 7

(i) What is the probability that the diameter of a randomly selected tree will be at least 10 in.?

(ii) What is the probability that the diameter of a randomly selected tree will exceed 20 in.?

(iii) What is the probability that the diameter of a randomly selected tree will be between 5 in and 10 in.?

14 a) The time (in hours) required to repair a machine is exponentially distributed with mean 2. (i) What is the probability that the repairing time exceeds 2 hours? (ii) What is the conditional probability that a repair takes at least 10 hours given that its duration exceeds 9 hours? 7

b) The joint probability density function of two continuous random variables X and Y is given by $f(x,y) = \begin{cases} kx^2y & if\ 1 \le x \le 4, 0 \le y \le 4 \\ 0 & otherwise \end{cases}$. **7**

Find (i) value of k (ii) $P[X \ge 2, Y \le 2]$ and (iii) $P[X + Y < 3]$.

Module -3

15 a) Assume that X(t) is a random process defined as follows: X(t) = A cos (2πt + ∅) where A is a zero-mean normal random variable with variance $\sigma_A^2 = 2$ and ∅ is uniformly distributed random variable over the interval −π ≤ φ ≤ π. A and φ are statistically independent. Let the random variable Y be defined as $Y = \int_0^1 X(t)dt$. Determine (i) the mean of Y (ii) the variance of Y. **7**

b) Show that the random process defined by $X(t) = A\sin(\alpha t + \theta)$, where A and α are constants and θ is a random variable uniformly distributed in $[0, 2\pi]$ is a wide sense stationary process. **7**

16 a) Determine the autocorrelation function of the random process with the power spectral density given by **7**

$$S_{XX}(w) = \begin{Bmatrix} S_O & |w| < w_0 \\ 0 & otherwise \end{Bmatrix}$$

b) Car arrive at a gas station according to a Poisson process at an average rate of 12 cars per hour. The station has only one attendant. If the attendant decides to take a 2-minute coffee break when there are no cars at the station. What is the probability that one or more cars will be waiting when he comes back from the break given that any car that arrives when he is on coffee break waits for him to get back? **7**

Module -4

17 a) Find the root of the equation $\cos x - xe^x = 0$ that lies between 0 and 1, using regula- falsi method, correct to four decimal places. **7**

b) Find the equation of the curve that passes through the points $(0, 2)$, $(1, 3)$, $(2, 12)$ and $(5, 147)$ by Lagrange's interpolation formula. Also find $y(3)$. **7**

18 a) Given a function $y = f(x)$ by the following table. Using Newton's interpolation formula, find $f(0.2)$. **7**

x	0	1	2	3	4	5	6
y	176	185	194	203	212	220	229

b) Evaluate $\int_0^1 \frac{dx}{1+x}$ using Simpson's one third rule. Find the error by comparing with actual integration up to four decimal places. [Take $h = \frac{1}{6}$]. 7

Module -5

19 a) Apply Gauss-Seidel method to solve the equations 7

$$20x + y - 2z = 17, \quad 3x + 20y - z = -18, \quad 2x - 3y + 20z = 25.$$

b) Given $\frac{dy}{dx} = x + y$, $y(0) = 1$. Using Euler's method, find $y(0.1), y(0.2)$ and $y(0.3)$ by taking $h = 0.1$. Hence obtain $y(0.4)$ using Adams- Moulton predictor- corrector method. 7

20 a) Given $y' = 1 + xy$, $y(0) = 2$. Find y at $x = 0.1$, using fourth order Runge-Kutta method, by taking $h = 0.1$. 7

b) By the method of least squares, find the straight line that best fits the following data. 7

x	1	2	3	4	5
y	14	27	40	55	68

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