Reg No.:_______________ Name:__________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Fourth Semester B.Tech Degree (S,FE) Examination June 2022 (2015 scheme)

Course Code: MA204

Course Name: PROBABILITY, RANDOM PROCESSES AND NUMERICAL METHODS (AE, EC)

Max. Marks: 100 Duration: 3 Hours

Normal distribution table is allowed in the examination hall.

PART A

Answer any two questions

1 a) The probability density function of a random variable X is

x	0	1	2	3	4	5	6
$f(x)$	k	$3k$	$5k$	$7k$	$9k$	$11k$	$13k$

Then find (i) k $(ii) P(X < 4)$ (iii) Mean (iv) Variance. 7

b) In a normal distribution 17% of the items are below 30 and 17% are above 60. Find the mean and standard deviation of the distribution. 8

2 a) If a random variable X is uniformly distributed with mean 1 and variance 3/4, find the probability that the random variable assumes only negative values. 7

b) If 5% of the electric bulbs manufactured by a company are defective, use Poisson distribution to find the probability that in a sample of 100 bulbs that (i) none is defective (ii) atleast 2 bulbs are defectives. 8

3 a) Fit a Binomial distribution to the following data: 7

x	0	1	2	3	4	5	6
f	4	20	27	12	7	6	4

b) The time (in hours) required to repair a machine is exponentially distributed with mean 2. What is the probability that the repair time exceeds 2 hours?

(i) What is the conditional probability that a repair takes atleast 10 hours given that its duration exceeds 9 hours? 8

PART B

Answer any two questions

4 a) If the joint distribution of (X,Y) is given by $f(x,y) = \dfrac{x+y}{21}, x = 1,2,3; y = 1,2$ 7

Then find the marginal distributions. Also find the means of X and Y.

b) If $X(t) = A\cos\lambda t + B\sin\lambda t$ where A and B are independent and normally distributed random variables with mean 0 and variance σ^2, obtain the covariance function of the process $\{X(t)\}$. 8

5 a) A random sample of size 100 is taken from a population whose mean is 80 and variance is 400. Using central limit theorem find with what probability can we assert that the sample mean will not differ from the population mean $\mu = 80$ by more than 6. 7

b) Find the power spectral density of a WSS random process X(t), whose autocorrelation function is $R(\tau) = \begin{cases} 1 - \frac{|\tau|}{T}, & |\tau| \le T \\ 0, & Otherwise \end{cases}$ 8

6 a) If $\{X(t)\}$ is a random process with mean $\mu(t) = 3$ and autocorrelation $R(t_1,t_2) = 9 + 4e^{-0.2|t_1 - t_2|}$, then find the mean, variance and covariance of the random variables X(5) and X(8). 7

b) If X and Y are random variables with joint pdf 8

$$f(x,y) = \begin{cases} k(x+2y) \ ; \ 0 < x < 1, \quad 0 < y < 1 \\ 0 \ ; \ elsewhere \end{cases}$$

Find k, the marginal distributions and check whether X and Y are independent.

PART C

Answer any two questions

7 a) If the transition probability matrix of a Markov chain is $P = \begin{bmatrix} 0 & 1 \\ 1/2 & 1/2 \end{bmatrix}$, then find the steady-state distribution of the chain. 5

b) A fair die is tossed repeatedly. If X_n denotes the maximum of numbers occurring in first n tosses, then find the transition probability matrix of the Markov chain $\{X_n\}$. 5

c) The following table gives the values of sin θ where θ is in degrees. Using Newton's interpolation formula estimate the value of (i) sin(8°) and (ii) sin (27°) 10

θ	5	10	15	20	25	30
Sinθ	0.0871	0.1736	0.2588	0.342	0.4226	0.5

8 a) A message transmission system is a markovian process with the transition

probability of the current message to the next message as given by the matrix $P = \begin{bmatrix} 0.2 & 0.3 & 0.5 \\ 0.1 & 0.2 & 0.7 \\ 0.6 & 0.3 & 0.1 \end{bmatrix}$ with initial probability [0.4 0.3 0.3]. Find the probabilities of the next message. 5

b) If the customers arrive at a bank according to a Poisson process with a mean rate of 3 per minute, then find the probability that during a time interval of 2 minutes (i) exactly 4 customers arrive and (ii) more than 4 customers arrive. 5

c) Evaluate $\int_0^2 xe^x dx$ using (i) Trapezoidal method and (ii) Simpson's 1/3 rule with 8 subintervals . 10

9 a) Find a positive solution of $1 + \cos x = 3x$ using Newton- Raphson method correct to 4 decimal places. 5

b) Find the value of $y(1.1)$ using Runge-Kutta method of fourth order given that $\frac{dy}{dx} = y^2 + xy$, $y(1) = 1, h = 0.1$ 5

c) The transition probability matrix of a Markov chain having three states 1,2 and 3 is $P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$ and the initial distribution is p(0) =[0.7 0.2 0.1]. 10

Find (i) $P[X_2= 3]$ and (ii) $P[X_3= 2, X_2= 3, X_1= 3, X_0= 2]$

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