Reg No.:_______________ 00000MA101121802 Name:_______________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B.Tech Degree S1 (S,FE) S2 (S) Examination May 2022 (2015 Scheme)

Course Code: MA101

Course Name: CALCULUS

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each carries 5 marks. Marks

1 a) Examine the convergence of $\sum_{k=1}^{\infty}\frac{1}{(\ln(k+1))^k}$ (2)

b) Find the Taylor series expansion of $f(x) = x\cos x$ at $x = \pi$., upto third degree term. (3)

2 a) Compute the total differential dz of $z=xe^{y^2}$. (2)

b) Find the slope of the surface $z=\sqrt{3x+2y}$ in the y-direction at the point (4,2) (3)

3 a) Find the velocity and acceleration at time t=2 of a particle moving along the curve $\mathbf{r}(t) = ti+\frac{1}{2}t^2j-\frac{1}{3}t^3k$. (2)

b) Find the normal to the surface yz+xz+xy = c at the point (-1,2,3). (3)

4 a) Evaluate $\int_0^1\int_0^1\int_0^1 e^{x+y+z}\,dx\,dy\,dz$ (2)

b) Evaluate $\iint_R \frac{\sin x}{x}\,dx\,dy$ where R is the triangular region bounded by the x axis y = x and x = 1. (3)

5 a) Find the value of constant c so that $\bar{F} = (3x-4y)\bar{\imath} + (cy-3z)\bar{\jmath} + (4y-5z)\bar{k}$ is solenoidal. (2)

b) The function $f(x,y) = xy + yz + zx$ is a potential function for the vector field $\bar{F}$. Find the vector field $\bar{F}$. (3)

6 a) Use divergence theorem to find the outward flux of the vector field $F(x,y,z) = 2x\,i + 3y\,j + z^2k$ across the unit cube bounded by the coordinate planes and the planes $x = 1, y = 1$ and $z = 1$. (2)

b) Using Greens Theorem evaluate $\oint_C y^2\,dx + x^2dy$,C is the square with vertices $(0,0), (1,0), (1,1)$ and $(0,1)$ (3)

PART B

Module 1

Answer any two questions, each carries 5 marks.

7 Find the interval of convergence and radius of convergence of $\sum_{k=1}^{\infty} \frac{(x-5)^k}{k^2}$ (5)

8 Check the convergence of the infinite series $\sum_{k=1}^{\infty} \frac{2^k k!}{k^k}$. (5)

9 Check the convergence of the alternating series $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}(k+3)}{k(k+1)}$. (5)

Module 1I

Answer any two questions, each carries 5 marks.

10 Let f be a differentiable function of one variable, and let w = f(u), where u = x+2y+3z.Show that $\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z} = 6\frac{dw}{du}$. (5)

11 Find the local linear approximation L(x ,y) of f(x ,y) $= \frac{1}{\sqrt{x^2+y^2}}$ at the point P(4,3).Compare the error in the approximation to f by L at the point Q(3.92,3.01) with the distance between P and Q. (5)

12 Locate all relative extrema and saddle points of $f(x,y) == 4xy - x^4 - y^4$ (5)

Module 1II

Answer any two questions, each carries 5 marks.

13 Find the directional derivative of $f(x,y) = \sqrt{xy}$ at (1, 4) in the direction of the unit vector that makes an angle $\pi/3$ with positive x-axis. (5)

14 A particle moves along a curve $x = 2t^2, y = t^2 - 4t, z = 3t - 5$ where t is the time. Find the component of acceleration at time t = 1 in the direction of $\vec{i} - 3\vec{j} + 2\vec{k}$ (5)

15 The motion of a particle is given by **r**(t)=t **i**+2t^2 **j**+3t **k**. Compute

a) Scalar and vector tangential component of acceleration at t=1. (5)

b) Scalar and vector normal component of acceleration at t=1

Module 1V

Answer any two questions, each carries 5 marks.

16 Use double integral to find the area of the region enclosed between the parabola $y^2 = -x$ and the line $4y - x = 5$. (5)

17 Evaluate $\int_0^1 \int_{4x}^4 e^{-y^2}\, dy\, dx$; by reversing the order of integration. (5)

18 Find the volume of the solid in the first octant bounded by the coordinate planes and the plane x+2y+z=6 (5)

Module V

Answer any three questions, each carries 5 marks.

19 If $\nabla\emptyset = 2xyz^3 i + x^2z^3 j + 3x^2z^2 yk$ find $\emptyset$ (5)

20 Use line integrals to find the area of the triangle with vertices $(0,0), (3,0), (0,2)$ (5)

21 Evaluate $\int_c xdx - yzdy + e^z dz$ where C is given by $x = t^3, y = -t, z = t^2, 1 \le t \le 2$ (5)

22 Find the work done by the force $\bar{F} = 2xy^3\ \bar{\imath} + (1 + 3x^2y^2)\bar{\jmath}$ along any path joining (0,0) and (1,2) (5)

23 Show that $\int_{(0,0)}^{(3,2)} 3x^2e^y dx + x^3e^y dy$ is independent of path. Hence evaluate $\int_{(0,0)}^{(3,2)} 3x^2e^y dx + x^3e^y dy$ (5)

Module VI

Answer any three questions, each carries 5 marks.

24 Using Gauss divergence theorem ,evaluate $\iint_s F.nds$ for $F = (x^2 - yz)i + (y^2 - xz)j + (z^2 - xy)k$ taken over the rectangular parallelepiped enclosed by $x = 0, x = a, y = 0, y = b,\ z = 0, z = c$ (5)

25 Use Stoke's theorem to evaluate $\int_c (yzdx + +zxdy + xydz)$ where C is the curve $x^2 + y^2 = 1,\ z = y^2$ (5)

26 Using Green's theorem evaluate $\int_C (xy + y^2)dx + x^2 dy$ where C is the boundary of the region bounded by $y = x^2$ and $x = y^2$ (5)

27 Find the mass of the lamina that is the portion of the cone $z = \sqrt{x^2 + y^2}$ between $z = 1$ and $z = 3$ if the density is $\emptyset(x, y, z) = x^2 z$ (5)

28 Evaluate the surface integral $\iint_\sigma f(x, y, z)\ ds$ where $f(x, y, z) = xy$, σ is the portion of the plane $x + y + z = 2$ in the first octant. (5)

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