APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

08 PALAKKAD CLUSTER

Q. P. Code: IAR0821111-I **(Pages: 2)** **Name:** .....

Reg. No: .....

FIRST SEMESTER M. TECH. DEGREE EXAMINATION DECEMBER 2021

Branch: Mechanical Engineering **Specialization:** Industrial Automation and Robotics

08ME6311 Advanced Mathematics and Optimization Techniques

Time: 3 Hours **Max. Marks: 60**

Answer all six questions.

Modules 1 to 6: Part 'a' of each question is compulsory and answer either part 'b' or part 'c' of each question.

Q.No.	Module 1	Marks
1.a	Write the vector v = (1,-2,5) as a linear combination of the vectors x = (1,1,1), y = (1,2,3) and z = (2,-1,1)	3
	Answer b or c	
b	Find the basis and dimension of the vector space V spanned by the vectors (2,-2,-4,1), (7,-7,-14,6), (3,-3,-6,2), (9,-9,-18,17)	6
c	Let W be the subset of R^3. Determine whether the subset W = { (x, y, z) : x + 3 y – 5 z = 0} is a subspace of R^3	6

Q.No.	Module 2	Marks
2.a	Let T be a linear operator on R^3 defined by T(x,y,z) = (2y + z, x – 4y ,3x).Find matrix representation of T relative to the basis {(1,1,1),(1,1,0),(1,0,0)}	3
	Answer b or c	
b	Let F: $R^4 \rightarrow R^3$ be the linear map defined by F(x, y, z, t) = (x – y + z + t, x + 2z - t , x + y + 3z – 3t). Find a basis and dimension of the image of F	6
c	Let F be the field of real numbers and let T be a linear map from F^3 to F^3 defined by T(x, y, z) = (x – y + 2z, 2x + y - z , -x – 2y). Check whether T is one-one.	6

Q.No.	Module 3	Marks
3.a	Using the standard inner product, verify triangle inequality for the vectors u = (-2,3,1) and v = (3,-4,1) in R^3.	3
	Answer b or c	
b	Check whether the following is an inner product in R^2: $\langle u,v \rangle = x_1 y_1 + 3x_2 y_2$ where $u = (x_1, x_2)$ and $u = (y_1, y_2)$	6

c	Find an orthonormal basis for the vector subspace of R^3 spanned by the vectors (2,0,1),(3,-1,5) and (0,4,2)	6

Q.No.	Module 4	
4.a	Write down the dual of the following problem. Maximise $z = 4x_1 + 2x_2$ subject to $-x_1 - x_2 \leq -3$, $-x_1 + x_2 \geq -2$ and $x_1, x_2 \geq 0$	3
	Answer b or c	
b	Using the Big M method find the Maximum value of $z = x_1 + 5x_2$ subject to $3x_1 + 4x_2 \leq 6$, $x_1 + 3x_2 \geq 2$ and $x_1, x_2 \geq 0$	6
c	Solve the following LPP using Simplex method. Maximise $Z = 2x_1 - x_2 + 2x_3$ subject to the constraints $2x_1 + x_2 \leq 10$, $x_1 + 2x_2 - 2x_3 \leq 20$, $x_2 + 2x_3 \leq 5$, x_1, x_2 & $x_3 \geq 0$	6

Q.No.	Module 5	
5.a	Explain Branch and bound method.	4
	Answer b or c	
b	Use branch and bound Technique to solve the following Max: $Z = 2x_1 + 2x_2$ such that $5x_1 + 3x_2 \leq 8$; $x_1 + 2x_2 \leq 4$; x_1, $x_2 \geq 0$ and integers.	8
c	Solve the following integer programming problem Min: $Z = x_1 - 3x_2$ such that $x_1 + x_2 \leq 5$; $-2x_1 + 4x_2 \leq 11$; x_1, $x_2 \geq 0$ and x_2 is an integer using Gomory's cutting plane method.	8

Q.No.	Module 6	
6.a	Obtain the necessary conditions for Lagrange multiplier method for the optimum solution of the following problem. Minimize $f(x,y) = 3e^{(2x+1)} + 2e^{(y+5)}$ subject to $g(x,y) = x + y - 7 = 0$	4
	Answer b or c	
b	Maximize $U = xy$ subject to: $x + y \leq 100$ and $x \leq 40$	8
c	Minimize $z = 3x_1^2 + 4x_2^2$ subject to $2x_1 - 3x_2 = 0$ by the Lagrange multiplier method	8