01MAT102052004 B

A **Pages: 3**

Reg No.:_____ Name:_____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Second Semester B.Tech Degree Examination July 2021

Course Code: MAT102

Course Name: VECTOR CALCULUS, DIFFERENTIAL EQUATIONS AND TRANSFORMS

(2019 Scheme)

AN Session

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each carries 3 marks.

1 Find the velocity and acceleration of a particle whose position vector is given by $\vec{r}(t) = e^t\,\vec{i} + e^{-t}\,\vec{j}$ *at* $t = 0$. (3)

2 If C is the unit circle $x^2 + y^2 = 1$ oriented counter clockwise, find $\int_C x\,dx + y\,dy$. (3)

3 Determine the sources and sinks of the vector field $\vec{f}(x,y,z) = 2\left(x^3 - 2x\right)\vec{i} + 2\left(y^3 - 2y\right)\vec{j} + 2\left(z^3 - 2z\right)\vec{k}$. (3)

4 Evaluate $\iint_\sigma x\,z\,dS$ where σ is the part of the plane $x + y + z = 1$ that lies in the first octant. (3)

5 Solve the initial value problem $y'' + y' - 2y = 0,\ \ y(0) = 4,\ \ y'(0) = -5$. (3)

6 Solve $y'''' + 10y'' + 9y = 0$. (3)

7 Find the Laplace transform of $e^{-3t}\cos 2t$. (3)

8 Find the inverse Laplace Transform of $\dfrac{1}{(s+a)(s+b)}$ (3)

9 Find the Fourier cosine transform of the function $f(x) = \begin{cases} x & if \quad 0 < x < a \\ 0 & if \quad x > a \end{cases}$ (3)

10 Using Fourier sine integral , show that

$$\int_0^\infty \frac{1-\cos \pi w}{w} \sin x\,w\,dw = \begin{cases} \dfrac{\pi}{2} & if\ 0 < x < \pi \\ 0 & if \quad x > \pi \end{cases}$$ (3)

PART B

Answer one full question from each module, each question carries 14 marks

Module-I

11 a) Find the directional derivative of $\varphi(x,y,z) = x^3z - yx^2 + z^2$ at (1,1,1) in the direction of $\vec{a} = 2\vec{i} - \vec{j} + 2\vec{k}$. Also find the maximum directional derivative. (7)

b) Show that the vector field $\vec{f}(x,y) = xy^2\,\vec{i} + x^2y\,\vec{j}$ is conservative and find ϕ such that $\vec{f} = \nabla\phi$. Hence evaluate $\int_{(1,2)}^{(2,4)} xy^2\,dx + x^2y\,dy$. (7)

12 a) Find the parametric equation of the tangent line to the graph $\vec{r}(t) = t^2\,\vec{i} - \frac{1}{t+3}\,\vec{j} + (4-t^2)\,\vec{k}$ at (4,-1,0). (7)

b) Using line integral evaluate $\int_C x^2y\,dx + x\,dy$ where C is the triangular path connecting (0,0) ,(1,0) and (1,2) in the positive direction. (7)

Module-II

13 a) Use Green's theorem to evaluate $\int_C \log(1+y)\,dx - \frac{xy}{1+y}\,dy$ where C is the triangle with vertices (0,0),(2,0) and (0,4). (7)

b) Use Divergence theorem to find the outward flux of the vector filed $\vec{F} = (2x + y^2)\,\vec{i} + xy\,\vec{j} + (xy - 2z)\,\vec{k}$ across the surface of the tetrahedron bounded by $x + y + z = 2$ and the coordinate planes. (7)

14 a) Find the flux of the vector field $\vec{F}(x,y,z) = x\,\vec{k}$ across the surface, the portion of the paraboloid $z = x^2 + y^2$ below the plane $z = 2y$ oriented by downward unit normal. (7)

b) Use Stokes theorem evaluate $\oint_C \vec{f}.dr$ where $\vec{f}(x,y,z) = (z-y)\,\vec{i} + (z+x)\,\vec{j} - (x+y)\,\vec{k}$ and C is the boundary of the paraboloid $z = 9 - x^2 - y^2$ above the XY-plane with upward orientation. (7)

Module-III

15 a) Solve using the method of undetermined coefficients $y'' - 4y' - 5y = 4\cos 2x$. (7)

b) Solve using the Method of variation of parameters $y'' + y = \text{cosec } x$ (7)

16 a) Solve using the method of undetermined coefficients (7)

$y''-7y'+12y=e^{2x}, \quad y(0)=1,\ y'(0)=2$.

b) Solve the boundary value problem $x^2y''-3xy'+4y=0, \quad y(1)=0,\ y'(1)=1$ (7)

Module-IV

17 a) Using convolution theorem find the Laplace inverse of $\dfrac{2}{\left(s^2+1\right)\left(s^2+25\right)}$ (7)

b) Using Laplace Transform solve $y''+2y'+y=e^{-t} \quad y(0)=0,\ y'(0)=1$ (7)

18 a) Express in terms of unit step function and hence find the Laplace Transform of

$$f(t)=\begin{cases} t^2, & if \quad 0<t<2 \\ t-1 & if \quad 2<t<3 \\ 7 & if \quad t>3 \end{cases}$$ (7)

b) Find the inverse Laplace Transform of $\dfrac{s^2+2}{s\left(s^2+9\right)}$ (7)

Module-V

19 a) Find the Fourier Transform of $f(x)=\begin{cases} |x| & if \quad -1<x<1 \\ 0 & otherwise \end{cases}$ (7)

b) Find the Fourier integral of $f(x)=\begin{cases} \pi-x & if \quad 0<x<\pi \\ 0 & otherwise \end{cases}$ (7)

20 a) Find the Fourier Sine Transform of $f(x)=\begin{cases} x & if \ 0<x<1 \\ 3-x & if \ 1<x<3 \\ 0 & if \ x>3 \end{cases}$ (7)

b) Represent $f(x)=e^{-kx} \quad x>0,\ k>0$ as a Fourier Cosine integral (7)

****