

**APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY**  
**SECOND SEMESTER M.TECH DEGREE EXAMINATION, OCT 2021**

*Branch: Electronics and Communication Engineering*

*Stream: Signal Processing*

*Course Code & Name: 01EC6306 Multirate Systems and Wavelets*

Answer any two full questions from each part

Limit answers to the required points.

Max. Marks: 60

Duration: 3 hours

**PART A**

1. a. Obtain the 2 band polyphase decomposition for the following IIR filter. 4

$$H(z) = \frac{1 - 2z^{-1}}{1 + 3z^{-1}}$$

- b. Develop an expression for the output  $y(n)$  as a function of the input  $x(n)$  for the multirate structure shown below. Write the relevant properties/identities used at various stages. 5



2. Consider the following multirate system shown in Fig. 1. The frequency response of the input signal,  $X(e^{j\omega})$  and filter response,  $H(e^{j\omega})$  are given in Fig. 2 and Fig. 3, respectively. Sketch the output response  $Y(e^{j\omega})$  for the given signals  $X(e^{j\omega})$  and  $H(e^{j\omega})$  by drawing the intermediate output signals at each point. 9

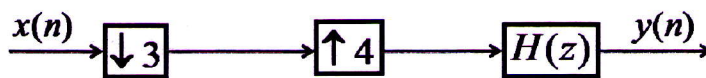


Fig. 1

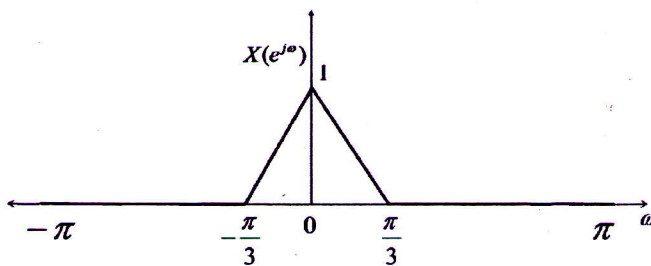


Fig. 2

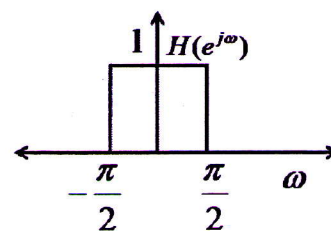


Fig. 3

3. a. In a two channel QMF bank, the polyphase components are  $E_0(z) = 1 + 0.5z^{-1} - 0.8z^{-2}$  and  $E_1(z) = 2 - 1.5z^{-1} + 2.3z^{-2}$ . Find the analysis and synthesis filters. You can assume a perfect reconstruction QMF bank. 6
- b. What are the common errors that can be created in QMF banks? 3

### PART B

4. a. What are the limitations of Fourier transform and short-time Fourier transform? How it is overcome in Wavelet transform? 5
- b. Find the two level Discrete Wavelet decomposition (with Mallat filter bank approach) for the sequence  $x(n) = \{4, 2, -2, 0, 6, 2, 5, 3\}$  using normalized Haar wavelet coefficients. 4
5. Design Daubechies Orthogonal Wavelet system with two vanishing moments using time domain approach. 9
6. a. Show that Haar scaling function and its translates are orthogonal. 3
- b. Explain the concept of nested space in wavelet theory. 3
- c. What is refinement relation? Give examples for refinement relations. 3

### PART C

7. Derive the analysis filter bank structure for a Biorthogonal Wavelet System starting from the basic two scale equations. 12
8. a. State the relationship between primal set and dual set in a biorthogonal wavelet system. Let vectors  $p_1 = (2, 0)$  and  $p_2 = (2, \sqrt{3})$  forms the primal set of a biorthogonal wavelet system. Find the dual set,  $[d_1, d_2]$  of the given system. Then show that the vectors  $p_1$  and  $p_2$  along with the vectors  $d_1$  and  $d_2$  form a biorthogonal system of vectors. 6
- b. Draw the Haar wavelet packet basis for three levels of decomposition. 6
9. Using filter bank approach, compute two level DWT on the 4X4 image whose pixel values are given below. Let  $h(k) = \left\{ \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\}$  and  $g(k) = \left\{ \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}} \right\}$  be the impulse response of analysis low-pass filter and analysis high-pass filter, respectively. 12

34	45	40	50
40	30	50	50
45	50	25	20
50	25	35	20