A **02000MA204052005** **Pages: 3**

Reg No.:_______________ Name:__________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Fourth Semester B.Tech Degree (S,FE) Examination August 2021 (2015 Scheme)

Course Code: MA204

Course Name: PROBABILITY, RANDOM PROCESSES AND NUMERICAL METHODS (AE, EC)

Max. Marks: 100 Duration: 3 Hours

Normal distribution table is allowed in the examination hall.

PART A

Answer any two questions

1 a) The probability distribution of a random variable X is

x	0	1	2	3
$f(x)$	c	$3c$	$5c$	c

(i) Find the value of c (ii) the distribution function

(iii) If $Y = X^2 + 2X$ then find $E(Y)$. 7

b) In a city, 4% of all licensed drivers will be involved in atleast one road accident in any given year. Use Poisson distribution to determine the probability that among 150 licensed drivers randomly chosen in this city 8

(i) only 5 will be involved in atleast one road accident in any given year.

(ii) Atmost 3 will be involved in atleast one road accident in any given year.

2 a) Derive the mean and variance of uniform distribution 7

b) In an intelligence test administered to 1000 children the average mark was 60 and the standard deviation was 20. Assuming the marks obtained follow Normal distribution, find the number of children who have scored (i) above 90 marks (ii) below 40 marks (iii) between 50 and 80 marks? 8

3 a) Fit a Binomial distribution to the following data: 7

x	0	1	2	3	4
f	5	29	36	25	5

b) The time required to repair a machine is exponentially distributed with a parameter 0.5. What is the probability that a repair time exceeds 2 hours? What is the conditional probability that a repair time takes atleast 10 hours given that its duration exceeds 9 hours? 8

PART B

Answer any two questions

4 a) A random sample of size 100 is taken from a population whose mean is 60 and variance is 400. Using Central Limit Theorem, find with what probability can we assert that the mean of the sample will not differ from $\mu = 60$ by more than 4? 7

b) The joint distribution of two random variables X and Y is given by $f(x, y) = \frac{x+y}{21}$, $x = 1, 2, 3$ and $y = 1, 2$. Find the marginal distributions of X and Y. Also find $E(X)$ and $E(Y)$. 8

5 a) Consider the random process with $X(t) = \cos(\omega t + \varphi)$ where φ is uniformly distributed random variable in $(-\pi, \pi)$. Check whether the process is stationary. 7

b) Obtain the power spectral density of the process $\{X(t)\}$ whose autocorrelation function $R(\tau) = ce^{-\alpha|\tau|}$ where $c > 0, \alpha > 0$. Also find the power of the process. 8

6 a) The joint probability density function of a two-dimensional random variable (X, Y) is given by $f(x, y) = xy^2 + \frac{x^2}{8}$, $0 \le x \le 2$, $0 \le y \le 1$.
Compute (i) $P(X > 1)$ (ii) $P(Y < \frac{1}{2})$ (iii) $P(X < Y)$ 7

b) $\{X(t) = A\cos \lambda t + B\sin \lambda t,\ t \ge 0\}$ is a random process where A and B are independent random variables following normal distribution with mean 0 and variance σ^2. Examine whether $\{X(t)\}$ is stationary. 8

PART C

Answer any two questions

7 a) A message transmission system is found to be Markovian with the transition probability of the current message to the next message as given by the matrix $P = \begin{bmatrix} 0.2 & 0.3 & 0.5 \\ 0.1 & 0.2 & 0.7 \\ 0.6 & 0.3 & 0.1 \end{bmatrix}$ with initial probability $[0.4 \quad 0.3 \quad 0.3]$. Find the probabilities of the next message. 7

b) The tpm of a Markov chain with states 1, 2, 3 is $P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$ and the initial distribution $P(0) = (0.7, 0.2, 0.1)$. 8
Find $(i) P(X_2) = 3$ $(ii) P(X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2)$

c) A radioactive source emits particle at the rate of 6 per minutes in accordance with Poisson process. Each particle emitted has a probability of $\frac{1}{3}$ being recorded. Find the probability that atleast 5 particles are recorded in 5 minutes. 5

8 a) Use Newton's forward difference formula to find y at $x = 1.5$. 7

x	0	1	2	3	4
y	7	10	13	22	43

b) Use Runge-kutta method to find $y(0.2)$ for the equation $\frac{dy}{dx} = \frac{y-x}{y+x}$ given $y(0) = 1$ (Take $h = 0.2$) 8

c) Find the approximate value of $\int_0^4 \sqrt{64 - x^2}\, dx$ by Trapezoidal rule (taking $h = 0.5$) 5

9 a) A man either drives a car or catches a train to go to office each day. He never goes 2 days in a row by train but if he drives one day, then the next day he is just as likely to drive again as he is to travel by train. Now suppose that on the first day of the week, the man tosses a fair dice and drove to work if and only if a 5 appears. Find (i) the probability that he takes train on the third day and (ii) the probability that he drives to work in the long run. 10

b) Find a positive root that lies between 0 and 1 of $3x = 1 + \cos x$ using Newton Raphson's method correct to 4 decimal places. 5

c) Given $f(0) = 1, f(1) = 3, f(3) = 55$. Use Lagrange's Interpolation method to find $f(2)$. 5

*****