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Reg No.:_____ Name:_____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Fourth Semester B.Tech Degree Examination July 2021 (2019 Scheme)

Course Code: MAT202
Course Name: PROBABILITY, STATISTICS AND NUMERICAL METHODS

Max. Marks: 100 Duration: 3 Hours

(Normal distribution table and t- distribution table are allowed)

PART A

(Answer all questions; each question carries 3 marks) Marks

1 Determine the binomial distribution for which mean is 4 and variance is 3. 3

2 X follows Poisson distribution with mean 6. Find $P(X = 1), Variance\ (X)$. 3

3 A continuous random variable X has PDF $f(x) = \frac{k}{1+x^2}$; $-\infty < x < \infty$ 3

Determine (i) k (ii) $P(X \geq 0)$

4 A random variable X follows exponential distribution with mean 3. Find $P(X > 3), Variance(X)$ 3

5 The proportion of a characteristic of a population is $p = 0.37$. Find the mean and variance of the sample proportion obtained from a sample of size 100. 3

6 A sample of size 49 is taken with mean 35 and standard deviation 11 from a population. Find the 99% confidence interval for the population mean. 3

7 Use trapezoidal rule to evaluate $\int_0^1 x^3\, dx$ considering five subintervals. 3

8 Find a root between 0 and 1 for $cosx = 3x - 1$ using Newton- Raphson method correct to 3 decimal places. 3

9 Use Rungi-Kutta method of second order to find $y(0.1)$ for $\frac{dy}{dx} = \frac{y^2-x^2}{y^2+x^2}$, $y(0) = 1$. (Take $h = 0.1$) 3

10 Given $\frac{dy}{dx} = 1 - y$, $y(0) = 0$. Use Euler's method with $h = 0.1$, to compute the value of $y(0.2)$. 3

PART B

(Answer one full question from each module, each question carries 14 marks)

Module -1

11 a) The pdf of a random variable X is given below. Find $k, P(X < 4), E(X), Var(X)$. 7

X	0	1	2	3	4	5	6
$f(x)$	k	$3k$	$5k$	$7k$	$9k$	$11k$	$13k$

b) Prove that binomial distribution can be approximated to Poisson distribution when n is large, p is small and $np = \lambda$. 7

12 a) A gambler plays a game of rolling a die with the following rules. He will win Rs. 200 if he throws a 6, but will lose Rs. 40 if he throws 4 or 5 and lose Rs. 20 if he throws 1, 2 or 3. Find the expected value that the gambler may gain. 7

b) The joint distribution of (X, Y) is given by $f(x, y) = \frac{x+y}{21}$, $x = 1,2,3$ and $y = 1, 2$. Then find the marginal distributions. Also, find $E(X)$, $E(Y)$. 7

Module -2

13 a) In an examination, 30% of the students got marks below 40 and 10% got marks above 75. Assuming the marks are normally distributed find, the mean and standard deviation of the distribution. 7

b) Buses arrive at a specified stop at 15 minutes interval starting at 8 am. If a passenger arrives at the stop at a random time that is uniformly distributed between 8.00 and 8.30 hours, find the probability that the passenger waits (i) less than 6 minutes for the bus (ii) atleast 12 minutes for the bus. 7

14 a) A distribution with unknown mean μ has variance 1.5. Use Central Limit Theorem to find, how large a sample should be taken from the distribution in order that the probability that the sample mean will be with in the 0.5 of the population mean is 0.95. 7

b) The joint PDF of (X, Y) is given by $f(x, y) = kxy \quad 0 < x < 4; \ 1 < y < 5$
$= 0 \quad$ elsewhere 7

Find value of k. Determine marginal pdf of X and Y. Evaluate $P[X \geq 3, Y \leq 2]$. Check whether X, Y are independent?

Module -3

15 a) A sample of 20 items has mean 42 and SD 5. Test whether the sample is from a population with mean 45? (5% level of significance) 7

b) The mean life time of certain products is 1800 hrs with SD of 100 hrs. By applying a new technique, it is claimed that the mean life has increased. To test the claim a sample of 50 products were taken and it is found that the mean life time is 1850 hrs. Can we support the claim at 1% level of significance? 7

16 a) In a university 325 out of 600 students are boys. Does this information support the conclusion that majority of students in this university are boys? (use 5% level of significance) 7

b) Random samples drawn from two countries gave the following data relating to height of adult males. 7

	Country A	Country B
Mean height	67.42	67.25
Standard deviation	2.58	2.50
Number in samples	1000	1200

Is the difference between the means significant? (5% level of significance)

Module -4

17 a) The population of a town in the census is as given in the data. Estimate the population in the year 1996 using Newton's backward interpolation formula. 7

Year (x) :	1961	1971	1981	1991	2001
Population (y) :	46	66	81	93	101

b) Using Lagrange's Interpolation method, find the polynomial $f(x)$ which agree with the following data: $f(1) = 1, f(3) = 27, f(4) = 64$. Hence find $f(2)$. 7

18 a) Using Newton's divided difference interpolation formula evaluate $f(3)$ from the following table: 7

x	1	2	4	5	6
y	14	15	5	6	19

b) Evaluate $\int_0^1 \frac{1}{1+x^2} dx$ using Simpson's 1/3 rd rule with $h = 0.1$. 7

Module -5

19 a) Obtain the value of $y(0.1)$ using Runge-Kutta method of fourth order for the differential Equation $dy/dx = -y$ and $y(0) = 1$. (Take $h = 0.1$) 7

b) Use the method of least squares to fit a straight line $y = ax + b$ for the following data: 7

x	1	2	3	4	5
y	6	7	9	10	12

20 Solve by Gauss-Siedel method correct to 3 decimal places. 14

$$10x - 5y - 2z = 3, \quad 4x - 10y + 3z = 3, \quad x + 6y + 10z = 3$$

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