Reg No.:_____ Name:_____

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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Fourth Semester B.Tech Degree Examination July 2021 (2019 Scheme)

Course Code: MAT208

Course Name: PROBABILITY,STATISTICS AND ADVANCED GRAPH THEORY

Max. Marks: 100 **Duration: 3 Hours**

PART A

(Answer all questions; each question carries 3 marks) Marks

1 If $f(x) = \frac{K}{2^x}$, is a probability mass function for random variable that can take on values 0,1,2,3,4 . Find K — 3

2 If X is a Poisson variable such that P (X= 2) = P (X = 3). Find mean and variance. — 3

3 The diameter of a cylindrically shaped cable is a random variable X with pdf $f(x) = \begin{cases} 6x(1-x) & if \quad 0 \le x \le 1 \\ 0 & otherwise \end{cases}$, Find the mean of the cable diameter. — 3

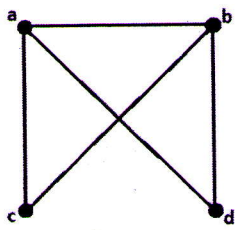
4 Random variable X is uniformly distributed in the interval (-k, k). Find k if $P(X \geq 1) = \frac{1}{3}$. — 3

5 Define the Type I ,Type II errors and Level of significance in Hypothesis testing . — 3

6 In a random sample of 500 people selected from the population of a city 60 were found to be left-handed. Find a 95% confidence interval for the proportion of left handed — 3

7 Does a simple graph that has five vertices each of degree 3 exist? Give reason — 3

8 Find the adjacency matrix representation of the graph G given below — 3



9 What is the chromatic number of each of the graphs K_5 , $K_{2,3}$, and W_5 — 3

10 How many edges does a full binary tree with 1000 internal vertices have? 3

PART B

(Answer one full question from each module, each question carries 14 marks)

Module -1

11 a) The following table gives the probability that a certain computer will malfunction 0,1,2,3,4 ,5,6 times on any one day. 7

x	0	1	2	3	4	5	6
f(x)	0.17	0.29	0.27	0.16	0.07	0.03	0.01

Find the mean and standard deviation of the probability function.

b) A company that produces fine crystal knows from experience that 10% of its goblets have cosmetic flaws and must be classified as "seconds." 7

Among six randomly selected goblets, what is the probability that

(i) only one is a second?

(ii) at least two are seconds?

12 a) Two fair dice are rolled. Let X denote the number on the first die, Y = 0 or 1, according as the first die shows an even number or odd number respectively. 7

Find (i)The joint probability distribution of X and Y.

(ii) The marginal pdf of X and Y

(iii) Are X and Y independent

b) Find the mean and variance of Poisson distribution 7

Module -2

13 a) If the probability function of a continuous random variable is given by 7

$$F(x) = \begin{cases} Kx^2 & 0 < x < 1 \\ 0 & otherwise \end{cases}$$

(i) Find K

(ii) $P(\frac{1}{4} < x < \frac{3}{4})$

(iii) $P(X > \frac{2}{3})$

b) In a Normal distribution 17% of items are below 30 and 17% of the items are above 60. Find mean and variance 7

14 a) Let X be the time between two successive arrivals at the drive up window of a local bank .If X has an exponential distribution with $\lambda = 1$,Compute 7

(i) The Distribution Function f(x)

(ii) $P(2 \leq x \leq 5)$

b) Consider two continuous random variable X and Y with joint pdf 7

$f(x, y) = 4xy, 0 \leq x \leq 1, 0 \leq y \leq 1.$

(i) Check whether f(x ,y) is a valid pdf

(ii) Calculate the marginal pdf of X and Y

Module -3

15 a) The mean length of life of a certain cutting tool is 41.5 hrs with standard deviation of 2.5 hrs. What is the probability that a simple random sample of size 50 drawn from the population will have a mean between 40.5 hrs and 42 hrs. 7

b) A manufacturer claims that his machine is producing bolts of which 8% is defective. A random sample of 400 is taken from a large consignment and is found to contain 30 defective bolts .Test the validity of his claim. 7

16 a) A manufacturer of sprinkler systems used for fire protection in office buildings claims that the true average system-activation temperature is 130°. A sample of 9 systems, when tested, yields a sample average activation temperature of 131.08°F. If the distribution of activation times is normal with standard deviation 1.5°F, does the data contradict the manufacturer's claim at significance level $\propto$ =0.01? 7

b) Two types of cars are compared for acceleration rate. 40 test runs are recorded for each car and the results for the mean elapsed time recorded below: 7

	Mean	Sample S.D
Car A	7.4	1.5
Car B	7.1	1.8

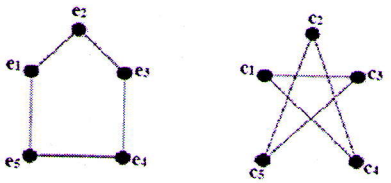
Determine if there is a difference in the mean elapsed times of the two car models at 95%confidence level.

Module -4

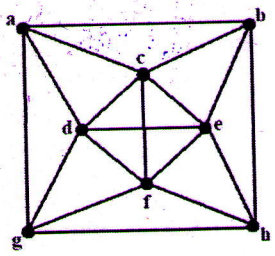
17 a) Prove that an undirected graph has an even number of vertices of odd degree. 7

b) Show that a Bipartite with n vertices has a maximum of $\frac{n^2}{4}$ edges 7

18 a) Define Isomorphism of Graphs. Determine whether the graphs given below are Isomorphic? 7



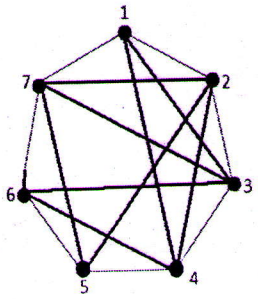
b) Define planar graphs. Determine whether this graph is planar. Justify your answer. 7



Module -5

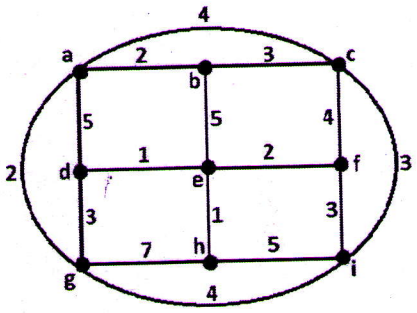
19 a) Show that a full m-ary tree with n vertices has $i = (n-1)/m$ internal vertices and $l = [(m-1)n+1]/m$ leaves, 5

b) Using the graph model given below, how can the final exam at a university be scheduled so that no student has two exams at the same time? 9



20 a) Show that a tree with n vertices has exactly n - 1 edges 7

b) Find a minimum spanning tree in the following weighted graph using Prim's algorithm 7



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