APJ ABDULKALAM TECHNOLOGICAL UNIVERSITY

08 PALAKKAD CLUSTER

Q. P. Code : IAR0820111-I (Pages: 2)

Name: .....

Reg. No: .....

FIRST SEMESTER M.TECH. DEGREE EXAMINATION MARCH 2021

Branch: Mechanical Engineering **Specialization: Industrial Automation and Robotics**

08ME6311 ADVANCED MATHEMATICS AND OPTIMIZATION TECHNIQUES

Time: 2 hour 15 minutes **Max. Marks: 60**

Answer all six questions.

Modules 1 to 6: Part 'a' of each question is compulsory and answer either part 'b' or part 'c' of each question.

Q.No.	Module 1	Marks
1.	Find a basis for the subspace S = { (a, b, c) : b = a – 3c} of R^3.	3
	Answer b or c	
b	Check whether the following matrices $\left\{\begin{bmatrix} 2 & 1 \\ -2 & -2 \end{bmatrix}, \begin{bmatrix} 3 & -3 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} -5 & 0 \\ 2 & 2 \end{bmatrix}, \begin{bmatrix} 3 & -3 \\ -2 & 2 \end{bmatrix}\right\}$ form a basis for the vector space of $M_{2\,X\,2}$ matrices.	6
c	Find the null space of the matrix A = $\begin{bmatrix} 1 & 0 & -2 & 6 \\ -3 & 6 & 6 & -6 \\ 2 & -3 & -4 & 6 \end{bmatrix}$ by finding a set of vectors that span Null(A).	6

Q.No.	Module 2	Marks
2. a	Let L : $P_1 \rightarrow P_2$ be defined by L[P(t)] = t P(t). Find the matrix L with respect to the bases S = {t, 1} and T = $\{t^2, t, 1\}$ for P_1 and P_2 , respectively. Using the matrix L, compute L[P(t)] if P(t) = 3t – 2.	3
	Answer b or c	
b	Let L: $R^4 \rightarrow R^3$ be defined by $L(a_1, a_2, a_3, a_4) = (a_1 + a_2,\ a_3 + a_4,\ a_1 + a_3)$. Find a basis for range L. Also verify Rank–Nullity theorem for the transformation L.	6
c	Determine whether the linear map T : $R^2 \rightarrow R^2$ defined by T(x, y) = (x – y, x – 2y) is nonsingular. Is T invertible ?	6

Q.No.	Module 3	Marks
3. a	Using the usual inner product, find a vector orthogonal to x = (–1, –1, 3) and y = (2, 3, –4) in R^3.	3

Answer b or c

b	Find an orthonormal basis for the vector subspace of R^3 spanned by the vectors (4, 0, 2),(3, −1, 5) and (0, 2, 1).	**6**
c	Check whether the following is an inner product in R^2: $\langle x, y \rangle = u_1 v_1 + 10u_2 v_2$ where $x = (u_1, u_2)$ and $y = (v_1, v_2)$.	**6**

Q.No.	**Module 4**	**Marks**
4. a	Solve the following problem graphically. Maximise $z = 60 x_1 + 40x_2$ subject to $2x_1 + x_2 \leq 60$, $x_1 \leq 25$, $x_2 \leq 35$ and $x_1, x_2 \geq 0$.	**3**

Answer b or c

b	Solve the following LPP using dual simplex method. Minimise $Z = 4x_1 + 2x_2$ subject to $x_1 + 2x_2 \geq 20$, $3x_1 + x_2 \geq 30$, $4x_1 + 3x_2 \geq 60$ and $x_1, x_2 \geq 0$.	**6**
c	Use the Simplex method to find the maximum value of $Z = 3x + 2y$ subject to $x + y \leq 4$, $x - y \leq 2$, and $x, y \geq 0$.	**6**

Q.No.	**Module 5**	**Marks**
5. a	What is meant by integer programming?	**4**

Answer b or c

b	Solve the following linear programming problem by Gomory's cutting plane technique. Max: $Z = 3x_2$ subject to the constraints $3x_1 + 2x_2 \leq 7$; $-x_1 + x_2 \leq 2$; $x_1, x_2 \geq 0$ and are integers.	**8**
c	Use branch and bound method to solve the following. Max: $Z = 6x_1 + 8x_2$ subject to the constraints $4x_1 + 16x_2 \leq 32$; $14x_1 + 4x_2 \leq 28$; $x_1, x_2 \geq 0$ and are integers.	**8**

Q.No.	**Module 6**	**Marks**
6. a	What do you understand by Quadratic programming? Explain briefly.	**4**

Answer b or c

b	Using Kuhn Tucker conditions Maximise: $Z = -x^2 - y^2 - z^2 + 4x + 6y$ subject to the constraints $x_1 + x_2 \leq 2$; $2x_1 + 3x_2 \leq 12$ with $x_1, x_2 \geq 0$.	**8**
c	Minimize $f = x^2 + y^2 + z^2$ subject to $2x + y - z = 1$ by the Lagrange multiplier method.	**8**