# APJ ABDULKALAM TECHNOLOGICAL UNIVERSITY

## 08 PALAKKAD CLUSTER

**Q. P. Code: CSE0820141-I** **(Pages: 4)**

**Name:** ...............................................

**Reg. No:** ..........................................

**FIRST SEMESTER M.TECH. DEGREE EXAMINATION MARCH 2021**

**Branch: Computer Science and Engineering** **Specialization: Computer Science and Engineering**

**08CS6041 MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE**

**(Common to CSE)**

**Time: 2 hour 15 minutes** **Max. Marks: 60**

**Answer all six questions.**

**Modules 1 to 6:** Part 'a' of each question is compulsory and answer either part 'b' or part 'c' of each question.

| Q. No. | Module 1 | Marks |
|---|---|---|
| 1. a | Find whether the following vectors are linearly independent or not? $e_1 = \begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix}$ $e_2 = \begin{bmatrix} -4 \\ 3 \\ 5 \end{bmatrix}$ $e_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ | 3 |
| | **Answer b or c** | |
| b | Find the LU decomposition of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 16 & 21 \\ 6 & 32 & 67 \end{bmatrix}$ | 6 |
| c | Compute the SVD of the matrix A= $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ | 6 |

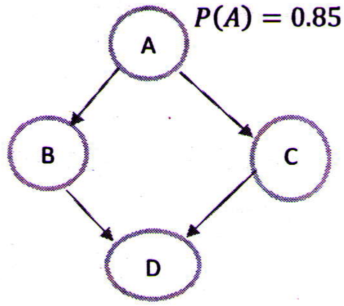
| Q. No. | Module 2 | Marks |
|---|---|---|
| 2. a | A box contains 6 apples and 4 oranges. Two are picked at random from the box at a time and found that one among them is orange. What is the probability that the other one is also orange? | 3 |
| | **Answer b or c** | |
| b | i) State Baye's Theorem | 2 |

ii) Find $P(D/A)$ based on the given diagram . 4

$P(A) = 0.85$

$P(B/A) = 0.3$

$P(B/\bar{A}) = 0.4$




$P(C/A) = 0.8$

$P(C/\bar{A}) = 0.1$

$P(D/B \cap C) = 0.3$

$P(D/B \cap \bar{C}) = 0.4$

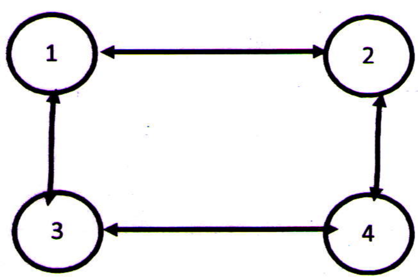
$P(D/\bar{B} \cap C) = 0.1$

$P(D/\bar{B} \cap \bar{C}) = 0.2$

**c** A binary communication channel carries data as of two types 0 and 1. Due to transmission error, sometimes a transmitted 0 is received as 1 and a transmitted one is received as zero. It is observed that the probability of receiving a 1 when transmitting a 1 is 0.90 and of receiving a 0 when transmitting a 0 is 0.95. Find the probability of 6

- (i) Receiving a '1'
- (ii) A 1 was transmitted given that 1 was received.
- (iii) A 0 was transmitted given that 0 was received.
- (iv) An error to occur.

| Q. No. | Module 3 | Marks |
|---|---|---|
| **3. a** | Define Ergodic Markov Chain? Check whether the given Markov Chain is Ergodic. Justify your answer. | 3 |




**Answer b or c**

**b** i) Define Basic Limit Theorem 2

ii) Find When stationary distribution is a limiting distribution for the Transition Probability Matrix given below 4

$$\begin{bmatrix} 0 & 1/2 & 1/2 \\ 1/4 & 1/4 & 1/2 \\ 0 & 2/3 & 1/3 \end{bmatrix}$$

c The transition probability matrix **6**

$$P=\begin{bmatrix}0.2 & 0.4 & 0.4\\0.6 & 0.2 & 0.2\\0.3 & 0.5 & 0.2\end{bmatrix}$$ and initial probability $P_0 = [0.7\ \ 0.2\ \ 0.1]$

Find the following

(i) $P(X_2 = 2)$

(ii) $P(X_3 = 2\ X_2 = 3\ X_1 = 3\ X_0 = 2)$

(iii) $P(X_4 = 3\ X_1 = 1)$

| Q. No. | Module 4 | Marks |
|---|---|---|
| 4. a | Define Birth death process , Poisson process and pure birth process | 3 |

**Answer b or c**

b Consider the CTMC X(t) with holding parameters $\lambda_1 = 3\ ,\lambda_2 = 4\ \lambda_3 = 5$. **6**

The transition probability matrix $P=\begin{bmatrix}0 & 1/2 & 1/2\\0 & 1/3 & 2/3\\1/2 & 0 & 1/2\end{bmatrix}$

Find the limiting distribution and the generator matrix.

c If the number of customers visiting the supermarket can be modelled by poisson distribution with λ= 20 customer/Hr. **6**

(i) Find the probability that there are four customers between 10am to 10.30 am.

(ii) Find the probability that there are 12 customers between 10am to 10.15 am and 8 customers between 10.15am to 11am

| Q. No. | Module 5 | Marks |
|---|---|---|
| 5. a | Consider $(M/M/1{:}\infty/FIFO)$ queuing system with λ=8/Hr and μ= 10/Hr. | 4 |

i. Find the probability that a customer has to wait more than 30 minutes to get his service completed

ii. Find the probability of atleast 10 customers in the system

**Answer b or c**

b The arrival of customers in a bank follows Poisson distribution with mean 15/Hr. Service time per customer is exponential with 12 minutes and the customer waiting area can accommodate maximum of three including the serving customer. Find the following **8**

i. Probability that the arriving customer will not wait

ii. Effective arrival rate

iii. Expected number of customers in the bank

iv. Expected waiting time of customers in the bank

v. Expected number of customers in the queue

vi. Expected waiting time of customers in the queue

| | | |
|---|---|---|
| c | A hospital has two doctors and four chairs in the waiting room. The patients who arrive at the hospital leave the hospital without consultation if all chairs are occupied. The patients arrive at an average rate of 6/Hr and the service time with an average of 4/Hr. If the system follows Markovian distribution. Find the following.<br>i. Probability that the arriving patient will not wait.<br>ii. Effective arrival rate<br>iii. Expected number of patients in the hospital.<br>iv. Expected waiting time of patients in the hospital.<br>v. Expected number of patients in the queue.<br>vi. Expected waiting time of patients in the queue. | 8 |

| Q. No. | Module 6 | Marks |
|---|---|---|
| 6. a | Find $E[x] = \begin{cases} \int_{-\infty}^{\infty} x\, f_X(x).\, d_x \\ \sum_{-\infty}^{\infty} x\, P_X(x) \end{cases}$<br>Where $f_X(x) = \begin{cases} \frac{1}{b+a} & a \le x \le b \\ 0 & \text{otherwise} \end{cases}$<br>$P_X(x) = \begin{cases} 1/4 & x=0 \\ 3/4 & x=2 \\ \text{i.} & \text{otherwise} \end{cases}$ | 4 |
| | **Answer b or c** | |
| b | Patients arrive at a hospital according to a Poisson distribution with the average arrival rate of 8 /Hr. It is estimated that the service time follows a random distribution with mean service time equal to 6 minutes and standard deviation equal to 15 minutes. Find $L_q$, $W_q$, $L_s$, $W_s$ | 8 |
| c | There are two counters in a car service station, one is for billing and the other is for serviced car delivery. The service station allow one customer to enter the station at a time and in each counter one is allow to stand. If the customer arrives according to Poisson process at a rate of 1/Hr and service time for each counter is 4/Hr and 2/Hr respectively.<br>i. Proportion of the customers who entered the car service station<br>ii. Average number of customers in the service station<br>iii. Average time the customer spends in the service station | 8 |