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Reg No.:_____ Name:_____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Fifth Semester B.Tech Degree Regular and Supplementary Examination December 2020

Course Code: CS309

Course Name: GRAPH THEORY AND COMBINATORICS

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each carries 3 marks. Marks

1 Prove that the number of vertices of odd degree in a graph is always even. (3)

2 11 children in a nursery school stand next to each other in circle such that they have different friends each time. How much such different arrangements are there? Justify your answer. (3)

3 What is the number of vertices in an undirected connected graph with 27 edges, 6 vertices of degree 2, 3 vertices of degree 4 and remaining of degree 3? (3)

4 Find the number of distinct hamiltonian cycles (not edge disjoint) possible in a complete graph of n vertices, where n>=3. (3)

PART B

Answer any two full questions, each carries 9 marks.

5 a) If graph G has 8 vertices and it is Eulerarian (has Euler Circuit), then find the maximum number of edges in the Graph G. (4)

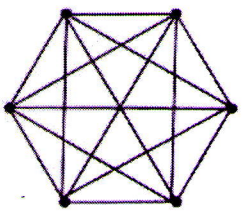
b) Prove that "A connected graph is Eulerian if and only if every vertex has even degree". (5)

6 a) State and prove Dirac's theorem for hamiltonicity. (5)

b) Consider an undirected graph G with 100 nodes. Find the minimum number of edges to be included in G so that the graph G is guaranteed to be connected. (4)

7 a) There are 20 people in a room. Suppose some pairs of the people shake hands and some don't. As the people leave the room you (who were not in the room) ask each person whether they shook hands an odd number of times or an even number of times. Prove that the number of people who answer "odd" is an even number. (4)

b) Label the vertices and edges in the graph given below, find and separately draw any four different Hamiltonian circuits contained in the given graph. (Note: The Hamiltonian circuits need not be edge-disjoint). (5)



PART C

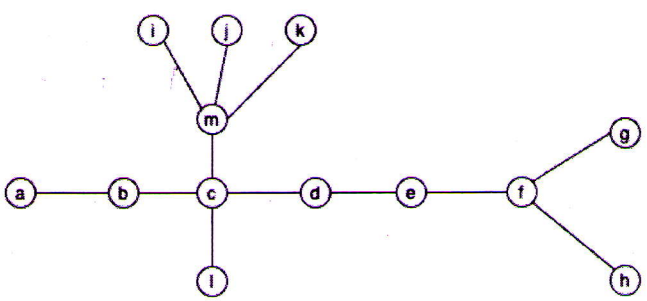
Answer all questions, each carries 3 marks.

8 Find the number of spanning trees in a complete graph of 4 labelled vertices. (3)

9 Prove the statement, "A graph with n vertices, n-1 edges and no circuits is connected". (3)

10 Prove that "Every cut set in a connected graph G must contain at least one branch of every spanning tree of G " (3)

11 Find the center and radius and of the tree shown below: (3)



PART D

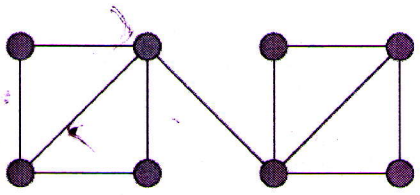
Answer any two full questions, each carries 9 marks.

12 a) Prove that a graph is a tree if and only if it is loop-less and has exactly one spanning tree (4)

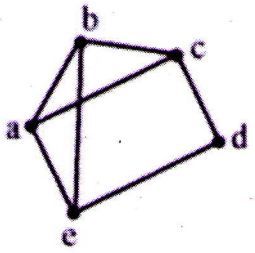
b) Define distance between the vertices d(v,u) of a connected graph G. Prove that the distance function of a connected graph is a metric. (5)

13 a) Define vertex and edge connectivity. Explain any one application showing the relevance of vertex and edge connectivity. (4)

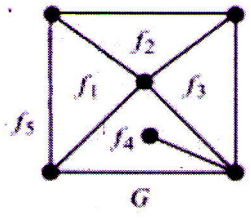
b) Define distance between two spanning trees of a graph. Draw any two spanning trees of the below graph such that the distance between them is >=3. (5)



14 a) Define a fundamental cut-set of a graph. Find all the fundamental cut-set of the graph below with respect to a spanning tree. (5)



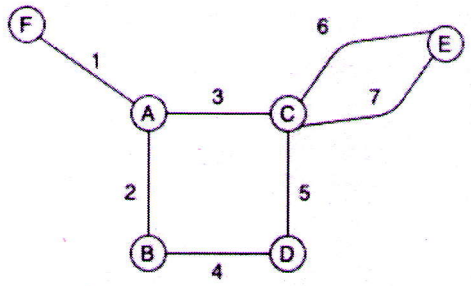
b) Draw the geometric dual of the graph G shown below (4)



PART E

Answer any four full questions, each carries 10 marks.

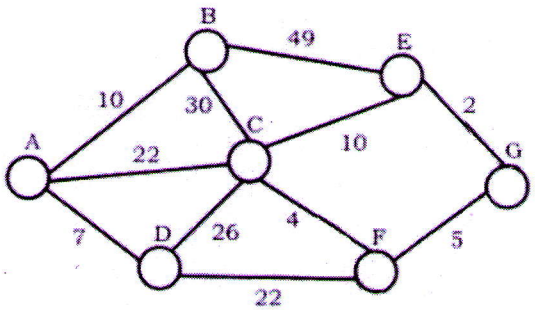
15 Consider the below graph. Find a spanning tree. Find the fundamental circuit matrix and fundamental cutset matrix with respect to the spanning tree. (10)



16 a) Draw the flowchart of connectedness and components algorithm. (5)

b) Prove that" If B is a circuit matrix of a connected graph G with e edges and n vertices then rank of B=e-n+1 (5)

17 Explain Prim's algorithm. Using Prim's algorithm construct a minimum spanning tree starting with node A. (10)



18 a) Adjacency matrix of a graph G is given below. From the given adjacency matrix construct a pictorial representation of the graph. Also list the properties of adjacency matrix representation of a graph. (6)

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

b) An undirected graph G has n nodes. its adjacency weight matrix is given by an n×n square matrix whose (i) diagonal elements are 0's and (ii) non-diagonal elements are 1's. State whether the following statements are TRUE/FALSE. Justify your answers (4)

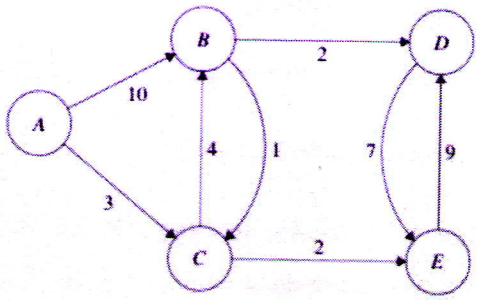
I. graph G has no minimum spanning tree

II. Graph G has unique MST of cost n−1

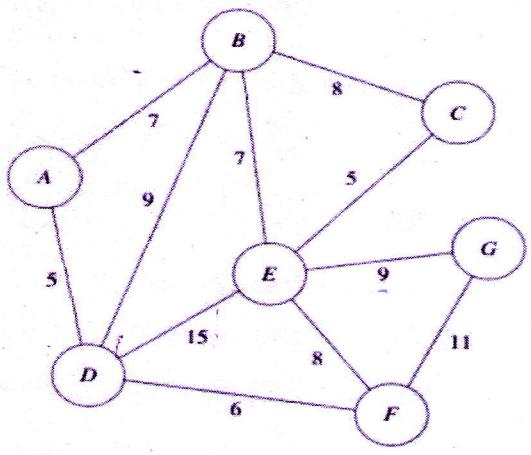
III. Graph G has multiple distinct MSTs, each of cost n−1

IV. Graph G has multiple spanning trees of different costs

19 Write Dijkstra's shortest path algorithm. Apply this algorithm to find the shortest paths from vertex A to all other vertices. (10)



20 Write Kruskal's algorithm for minimum spanning tree (MST). Apply this algorithm to find a MST of the below graph. (10)



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