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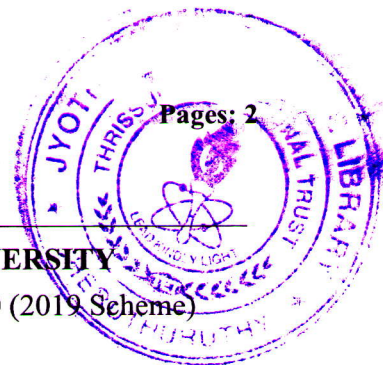
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Reg No.: _____

Name: _____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Third Semester B.Tech Degree Examination December 2020 (2019 Scheme)



Course Code: MAT201

Course Name: Partial Differential equations and Complex analysis

Max. Marks: 100

Duration: 3 Hours

PART A*Answer all questions. Each question carries 3 marks*

Marks

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|----|---|-----|
| 1 | Derive a partial differential equation from the relation $z = (x + y) f(x^2 - y^2)$ | (3) |
| 2 | Solve using direct integration $\frac{\partial^2 u}{\partial x \partial t} = e^{-t} \cos x$ | (3) |
| 3 | Solve $2z = xp + yq$. | (3) |
| 4 | Write any three assumptions in deriving one dimensional heat equation. | (3) |
| 5 | Show that an analytic function $f(z) = u + iv$ is constant if its real part is constant. | (3) |
| 6 | Show that the function $u = \sin x \cos hy$ is harmonic. | (3) |
| 7 | Find the Maclaurin series of $f(z) = \sin z$ | (3) |
| 8 | Evaluate $\oint_C \ln z dz$, where C is the unit circle $ z = 1$. | (3) |
| 9 | Find all singular points and residue of the function $\operatorname{cosec} z$ | (3) |
| 10 | Determine the location and order of zeros of the function $\sin^4\left(\frac{z}{2}\right)$ | (3) |

PART B*Answer any one full question from each module. Each question carries 14 marks***Module 1**

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|--------|--|-----|
| 11 (a) | Form the Partial differential equation by eliminating the arbitrary constants from $(x - a)^2 + (y - b)^2 = z^2 \cot^2 \alpha$ | (5) |
| (b) | Solve $2xz - px^2 - 2qxy + pq = 0$ | (9) |
| 12 (a) | Solve $\frac{\partial^3 z}{\partial^2 x \partial y} = \cos(2x + 3y)$ | (7) |
| (b) | Solve $x^2(y - z)p + y^2(z - x)q = z^2(x - y)$ | (7) |

Module 2

- | | | |
|--------|---|-----|
| 13 (a) | Derive the solution of the one dimensional wave equation $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ using variable separable method. | (6) |
| (b) | An insulated rod of length l has its ends A and B maintained at 0°C and | |

100° C respectively until steady state conditions prevail. If B is suddenly reduced to 0° C and maintained at 0° C, find the temperature at a distance x from A at time t . (8)

- 14 (a) Derive the one dimensional heat flow equation. (6)
- (b) A tightly stretched string of length l with fixed ends is initially in equilibrium position. If it is set vibrating by giving each points a velocity $v_0 \sin^3\left(\frac{\pi x}{l}\right)$. Find the displacement $y(x,t)$. (8)

Module 3

- 15 (a) Find an analytic function whose real part is $u = \sin x \cosh y$ (7)
- (b) Find the image of the strip $\frac{1}{2} \leq x \leq 1$ under the transformation $w = z^2$ (7)
- 16 (a) Check whether $w = \log z$ is analytic. (8)
- (b) Show that under the transformation $w = \frac{1}{z}$, the circle $x^2 + y^2 - 6x = 0$ is transformed into a straight line in the W plane. (6)

Module 4

- 17 (a) Integrate counter clockwise around the unit circle $\oint_C \frac{\sin 2z}{z^4} dz$ (7)
- (b) Find the Taylor series of $\frac{1}{1+z}$ about the centre $z_0 = i$ (7)
- 18 (a) Evaluate $\int_0^{1+i} (x - y + ix^2) dz$ along the parabola $y = x^2$. (7)
- (b) Evaluate $\oint_C \frac{\log z}{(z-4)^2} dz$ counter clockwise around the circle $|z - 3| = 2$. (7)

Module 5

- 19 (a) Find the Laurent's series expansion of $\frac{z^2-1}{z^2-5z+6}$ about $z=0$ in the region $2 < |z| < 3$ (5)
- (b) Evaluate $\int_0^{2\pi} \frac{d\theta}{\sqrt{2} - \cos\theta}$. (9)
- 20 (a) Evaluate $\oint_C \frac{z-23}{z^2-4z-5} dz$ where $C : |z - 2 - i| = 3.2$ using Residue theorem. (5)
- (b) Evaluate $\int_0^\infty \frac{(x^2+2)dx}{(x^2+1)(x^2+4)}$. (9)
