

D 30939

(Pages : 2)



SEVENTH SEMESTER B.TECH. (ENGINEERING) DEGREE  
EXAMINATION, DECEMBER 2003

CS2K. 703. NUMBER THEORY AND CRYPTOGRAPHY

Time : Three Hours

Maximum : 100 Marks

Part A

Answer all questions.

1. Let  $a$  and  $b$  be integers, not both zero. Then show that  $a$  and  $b$  are relatively prime if and only if there exist integers  $x$  and  $y$  such that  $ax + by = 1$ .
2. If  $p$  is prime and  $p$  divides  $a_1, a_2, a_3, \dots, a_n$ , then show that  $p$  divides  $a_k$  for some  $k$  where  $1 \leq k < n$ .
3. Show that for  $ax + by = c$ , if  $x_0, y_0$  is a particular solution then all solutions are of the form  $x = x_0 + (b/d)t$  and  $y = y_0 - (a/d)t$  for varying integer  $t$  and  $d = \gcd(a, b)$ .
4. Show that Wilson's theorem is true for  $p = 13$ .
5. Classify the security services and explain them briefly.
6. Tabulate the different types of attacks on encrypted messages.
7. Summarise the important aspects of conventional and public-key encryption.
8. What is Kerberos? What are the requirements for Kerberos?

(8 × 5 = 40 marks)

Part B

9. (a) (i) Show that if  $n$  is positive integer and  $\gcd(a, n) = 1$ , then  $a^{\phi(n)} \equiv 1 \pmod{n}$  and hence deduce Fermat's theorem  $a^{p-1} \equiv 1 \pmod{p}$  for a prime  $p$  and  $p$  does not divide  $a$ . (10 marks)
- (ii) Show that for any positive integer  $n \geq 1$ ,  $n = \sum_{d|n} \phi(d)$ , the sum being extended over all positive divisors of  $n$ . (5 marks)

Or

- (b) (i) Show that  $\gcd(a, b) \cdot \text{lcm}(a, b) = ab$  and hence show that  $\text{lcm}(a, b) = ab$  if and only if  $\gcd(a, b) = 1$ . (6 + 1 = 7 marks)
- (ii) Show that if  $2^k - 1$  is prime ( $k > 1$ ) then  $n = 2^{k-1}(2^k - 1)$  is perfect and every perfect number is of this form. (8 marks)

10. (a) (i) Solve  $172x + 20y = 1000$ . (8 marks)

- (ii) State and prove Wilson's theorem. (7 marks)

Or

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11. (a) Discuss in detail the simplified DES scheme illustrating the key generation and Encryption schemes.

Or

- (b) Discuss in detail the working of DES decryption algorithm and explain the avalanche effect in DES.

(15 marks)

12. (a) List and explain any *two* types of functions that may be used to produce an authenticator.

Or

- (b) Write the Secure Hash Algorithm explaining its working.

(15 marks)

[4 × 15 = 60 marks]