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THIRD SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, DECEMBER 2003

CS2K 301 / PTCS2K / IT2K. ENGINEERING MATHEMATICS—III

Time : Three Hours Maximum : 100 Marks

Answer **all** *questions.*

1. (a) Determine whether the set of vectors (2, 3, – 1), (3, 2, 2), (4, 4, – 1) in R^3 are linearly dependent or linearly independent.

   (b) Given the transformation $T : R^2 \to R^2$, where $T(x, y) = (ax + cy, cx + by)$, find $T^2(x, y)$.

   (c) Determine the rank of the matrix $\begin{pmatrix} 2 & -1 & 3 \\ 1 & -2 & 3 \\ 5 & 0 & 3 \end{pmatrix}$.

   (d) If $f(x) = \dfrac{x}{x+4}$, compute $f(A)$ for the matrix $A = \begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & -2 \\ -2 & 2 & 3 \end{pmatrix}$.

   (e) If $w = f(z)$ is an analytic function, prove that $\dfrac{dw}{dz} = \dfrac{\partial w}{\partial x} = -i\dfrac{\partial w}{\partial y}$.

   (f) Find the invariant points of the transformation $w = \dfrac{2z+6}{z+7}$.

   (g) Find the zeros of the function $\dfrac{z^3 - 1}{z^2 + 1}$ and indicate their order.

   (h) Using Cauchy's integral formula, evaluate $\int_C \dfrac{\cos \pi z}{z-1}\, dz$. C : the square with vertices $\pm 2 \pm 2i$.

   (8 × 5 = 40 marks)

2. (a) Find the subspace U of R^3 spanned by the vectors $v_1 = (1, -2, 1)$, $v_2 = (-2, 0, 3)$ and $v_3 = (3, -2, -2)$. Are the vectors (4, – 4, – 1) and (6, – 6, – 2) in U ?

   (b) Show that every set of n linearly independent vectors $\{v_1, v_2, \ldots, v_n\}$ of an n-dimensional vector space V is a basis of V.

   Or

Turn over

(c) Prove that the Schwarz inequality is equivalent to the triangle inequality $\|u+v\| \leq \|u\| + \|v\|$.

(d) The matrix $A = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$ is a linear transformation from R^3 into R^2. What is the image of $\begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}$ under A ?

3. (a) Show that if A and B are $n \times n$ matrices of rank n, then both AB and BA are of rank n.

(b) Find the characteristic values and the corresponding characteristic vectors of the matrix equation $(A - \lambda B)X = 0$ if $A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 3 \end{pmatrix}$.

Or

(c) For the matrix $A = \begin{pmatrix} 1 & 0 & 3 \\ 1 & -1 & 1 \\ -1 & 3 & 3 \end{pmatrix}$, find a pair of matrices (P, Q) such that PAQ is a diagonal matrix.

(d) If $A = \begin{bmatrix} 4 & 2 \\ -3 & -1 \end{bmatrix}$, use Cayley-Hamilton theorem to express A^n in terms of A and I.

4. (a) If $u(x, y)$ and $v(x, y)$ are harmonic functions in a region R, prove that the function $\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) + i\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$ is an analytic function of $Z = x + iy$.

(b) Determine the linear transformation that will map the points $1 - 2i$, $2 + i$, $2 + 3i$ respectively into $2 + 2i$, $1 + 3i$, 4. Find the invariant points of the transformation.

Or

(c) Construct the analytic function whose imaginary part is $e^{-x}(x \cos y + \sin y)$ and which equals 1 at the origin.

(d) Find the transformation that maps a wedge of angle $\frac{\pi}{6}$ of the w-plane onto the upper half of the Z-plane.

5. (a) Evaluate $\int_C (z^2 + 2z)\, dz$, where C is the

(i) upper half of the circle $|z - 1| = 1$.

(ii) lower half of the circle $|z - 1| = 1$.

(iii) Circle $|z - 1| = 1$.

(b) Finding a suitable Laurent's expansion for $\frac{1}{z^2\,(z - i)}$, find the residue of $f\,(z)$ at its singular points.

Or

(c) Using Cauchy's integral formula, evaluate $\int_C \frac{z^4}{(z + 1)\,(z - i)^2}\, dz$, where C is the ellipse $9x^2 + 4y^2 = 36$.

(d) By Contour integration show that $\int_0^\infty \frac{x^6}{(x^4 + 1)^2}\, dx = \frac{3\pi}{8\sqrt{2}}$.

(4 × 15 = 60 marks)