

D 30335

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THIRD SEMESTER B.TECH. (ENGINEERING) DEGREE
EXAMINATION, DECEMBER 2003

CS2K 301
PTCS2K / IT2K ENGINEERING MATHEMATICS - III

Time : Three Hours

Maximum : 100 Marks

Answer all questions.

1. (a) Determine whether the set of vectors $(2, 3, -1), (3, 2, 2), (4, 4, -1)$ in \mathbb{R}^3 are linearly dependent or linearly independent.

(b) Given the transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$, where $T(x, y) = (ax + cy, cx + by)$, find $T^2(x, y)$.

(c) Determine the rank of the matrix $\begin{pmatrix} 2 & -1 & 3 \\ 1 & -2 & 3 \\ 5 & 0 & 3 \end{pmatrix}$.

(d) If $f(x) = \frac{x}{x+4}$, compute $f(A)$ for the matrix $A = \begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & -2 \\ -2 & 2 & 3 \end{pmatrix}$.

(e) If $w = f(z)$ is an analytic function, prove that $\frac{dw}{dz} = \frac{\partial w}{\partial x} = -i \frac{\partial w}{\partial y}$.

(f) Find the invariant points of the transformation $w = \frac{2z+6}{z+7}$.

(g) Find the zeros of the function $\frac{z^3-1}{z^2+1}$ and indicate their order.

(h) Using Cauchy's integral formula, evaluate $\int_C \frac{\cos \pi z}{z-1} dz$. C : the square with vertices $\pm 2 \pm 2i$.

(8 × 5 = 40 marks)

2. (a) Find the subspace U of \mathbb{R}^3 spanned by the vectors $v_1 = (1, -2, 1), v_2 = (-2, 0, 3)$ and $v_3 = (3, -2, -2)$. Are the vectors $(4, -4, -1)$ and $(6, -6, -2)$ in U ?

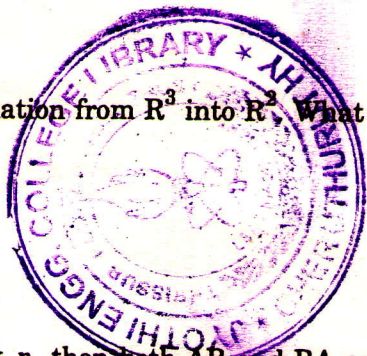
(b) Show that every set of n linearly independent vectors $\{v_1, v_2, \dots, v_n\}$ of an n -dimensional vector space V is a basis of V .

Or

Turn over

(c) Prove that the Schwarz inequality is equivalent to the triangle inequality $\|u + v\| \leq \|u\| + \|v\|$.

(d) The matrix $A = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$ is a linear transformation from \mathbb{R}^3 into \mathbb{R}^2 . What is the image of $\begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}$ under A ?



3. (a) Show that if A and B are $n \times n$ matrices of rank n , then both AB and BA are of rank n .

(b) Find the characteristic values and the corresponding characteristic vectors of the matrix

$$\text{equation } (A - \lambda B)X = 0 \text{ if } A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}, B = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 3 \end{pmatrix}.$$

Or

(c) For the matrix $A = \begin{pmatrix} 1 & 0 & 3 \\ 1 & -1 & 1 \\ -1 & 3 & 3 \end{pmatrix}$, find a pair of matrices (P, Q) such that PAQ is a diagonal matrix.

(d) If $A = \begin{bmatrix} 4 & 2 \\ -3 & -1 \end{bmatrix}$, use Cayley-Hamilton theorem to express A^n in terms of A and I .

4. (a) If $u(x, y)$ and $v(x, y)$ are harmonic functions in a region R , prove that the function $\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) + i\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$ is an analytic function of $Z = x + iy$.

(b) Determine the linear transformation that will map the points $1 - 2i, 2 + i, 2 + 3i$ respectively into $2 + 2i, 1 + 3i, 4$. Find the invariant points of the transformation.

Or

(c) Construct the analytic function whose imaginary part is $e^{-x}(x \cos y + \sin y)$ and which equals 1 at the origin.

(d) Find the transformation that maps a wedge of angle $\frac{\pi}{6}$ of the w -plane onto the upper half of the Z -plane.

5. (a) Evaluate $\int_C (z^2 + 2z) dz$, where C is the

- (i) upper half of the circle $|z - 1| = 1$.
- (ii) lower half of the circle $|z - 1| = 1$.
- (iii) Circle $|z - 1| = 1$.

(b) Finding a suitable Laurent's expansion for $\frac{1}{z^2(z-i)}$, find the residue of $f(z)$ at its singular points.

Or

(c) Using Cauchy's integral formula, evaluate $\int_C \frac{z^4}{(z+1)(z-i)^2} dz$, where C is the ellipse $9x^2 + 4y^2 = 36$.

(d) By Contour integration show that $\int_0^{\infty} \frac{x^6}{(x^4 + 1)^2} dx = \frac{3\pi}{8\sqrt{2}}$.

(4 × 15 = 60 marks)