(3 Pages)

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Name....................................

Reg. No.................................

COMBINED FIRST AND SECOND SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, JUNE 2003

EN.2K 102—MATHEMATICS—II

(New Scheme)

Time : Three Hours

Maximum : 100 Marks

Answer **all** *the questions.*

1. (a) Solve : $\dfrac{dy}{dx} = \dfrac{xy}{x^2 + y^2}$.

(b) Solve : $(D - 3D + 2)y = \sin 3x$.

(c) Find $L\left(\dfrac{e^{-at} - e^{-bt}}{t}\right)$.

(d) Find $L^{-1}\left[\dfrac{1}{(s+1)(s^2+2s+2)}\right]$.

(e) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$

(f) If $\vec{F} = (x^2 - y^2 + 2xz)i + (xz - xy + yz)j + (z^2 + x^2)k$, find Curl $\vec{F}$ and Curl Curl $\vec{F}$.

(g) Evaluate $\iint xy\, dx\, dy$ over the positive quadrant of the circle $x^2 + y^2 = a^2$.

(h) By using Green's theorem, evaluate $\int_c (x^2 y\, dx + y^3 dy)$ where C is the closed path formed by $y = x$ and $y^3 = x^2$ from (0, 0) to (1, 1).

(8 × 5 = 40 marks)

2. (a) (i) Show that the system of confocal parabolas $y^2 = 4a(x + a)$ is self-orthogonal.

(7 marks)

(ii) Solve : $(x^2 D^2 + 4xD + 2)y = e^x$.

(8 marks)

Or

Turn over

(b) (i) Solve : $2x\frac{dy}{dx} + y = 2x^3$. (6 marks)

(ii) By the method of variation of parameters, solve $\frac{d^2y}{dx^2} + 4y = \tan 2x$. (9 marks)

3. (a) (i) Evaluate $\int_0^\infty e^{-x^4} dx$, by using Gamma function. (5 marks)

(ii) $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = 4e^{2t}$ with $y(0) = -3$; $\frac{dy}{dt}(0) = 5$ by using Laplace transform techniques. (10 marks)

Or

(b) (i) Find (1) $L^{-1}\left[\frac{1}{s(s-a)}\right]$; and (2) $L^{-1}\left[\log\frac{1-s^2}{s^2}\right]$. (5 marks)

(ii) Solve the system of equations $\frac{dy}{dt} + ay = x$; $\frac{dx}{dt} + ax = y$ with $x(0) = 0$; $y(0) = 1$. (10 marks)

4. (a) (i) Find the directional derivative of $\phi = x^2yz + 4xz^2$ at $(1, -2, -1)$ in the direction of $2i - j - 2k$. (7 marks)

(ii) Prove that $\nabla \cdot \left(\vec{u} \times \vec{v}\right) = \vec{v} \cdot \left(\nabla \times \vec{u}\right) - \vec{u} \cdot \left(\nabla \times \vec{v}\right)$. (8 marks)

Or

(b) (i) Show that $\vec{F} = \left(6xy + z^3\right)$ if $\left(3x^2 - z\right)j + \left(3xz^2 - y\right)k$ is irrotational and hence find its scalar potential. (8 marks)

(ii) Prove that Curl $\left[f(r)\vec{r}\right] = 0$ if $\vec{r}$ is the position vector and r is its magnitude. (7 marks)

5. (a) (i) By changing the order of integration, evaluate $\int_0^1 \int_{x^2}^{2-x} xy\, dx\, dy$. (6 marks)

(ii) Evaluate $\iint_S \vec{F} \cdot \hat{n}\, ds$ if $\vec{F} = (x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k$ and S is the surface of $0 \le x \le a,\ 0 \le y \le b,\ 0 \le y \le c$.

(9 marks)

Or

(b) (i) Evaluate $\int_C \vec{F} \cdot d\vec{r}$ if $\vec{F} = (5xy - 6x^2)i + (2y - 4x)j$ and C is the curve in the *xy*-plane, $y = x^3$ from (1, 1) to (2, 8).

(ii) Verify Stoke's theorem for $\vec{F} = (x^2 + y^2 - 4)i + 3xyj + (2xz + z^2)k$ where S is the surface of the hemisphere $x^2 + y^2 + z^2 = 16$, above the *xy*-plane and C is its boundary.

(8 marks)