

C 26938

(3 Pages)

Name.....

Reg. No.....

COMBINED FIRST AND SECOND SEMESTER B.TECH. (ENGINEERING)
DEGREE EXAMINATION, JUNE 2003

EN.2K 102—MATHEMATICS—II

(New Scheme)

Time : Three Hours

Maximum : 100 Marks

Answer **all** the questions.

1. (a) Solve : $\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$.

(b) Solve : $(D - 3D + 2)y = \sin 3x$.

(c) Find $L\left(\frac{e^{-at} - e^{-bt}}{t}\right)$.

(d) Find $L^{-1}\left[\frac{1}{(s+1)(s^2+2s+2)}\right]$.

(e) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$.

(f) If $\vec{F} = (x^2 - y^2 + 2xz)\vec{i} + (xz - xy + yz)\vec{j} + (z^2 + x^2)\vec{k}$, find $\text{Curl } \vec{F}$ and $\text{Curl } \text{Curl } \vec{F}$.

(g) Evaluate $\iint xy \, dx \, dy$ over the positive quadrant of the circle $x^2 + y^2 = a^2$.

(h) By using Green's theorem, evaluate $\int_C (x^2 y \, dx + y^3 \, dy)$ where C is the closed path formed by $y = x$ and $y^3 = x^2$ from $(0, 0)$ to $(1, 1)$.

(8 × 5 = 40 marks)

2. (a) (i) Show that the system of confocal parabolas $y^2 = 4a(x + a)$ is self-orthogonal.

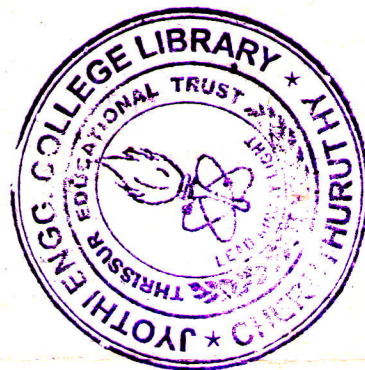
(7 marks)

(ii) Solve : $(x^2 D^2 + 4x D + 2)y = e^x$.

(8 marks)

Or

Turn over



(b) (i) Solve : $2x \frac{dy}{dx} + y = 2x^3$.

(6 marks)

(ii) By the method of variation of parameters, solve $\frac{d^2y}{dx^2} + 4y = \tan 2x$.

(9 marks)

3. (a) (i) Evaluate $\int_0^\infty e^{-x^4} dx$, by using Gamma function.

(5 marks)

(ii) $\frac{d^2y}{dt^2} - 3 \frac{dy}{dt} + 2y = 4e^{2t}$ with $y(0) = -3$; $\frac{dy}{dt}(0) = 5$ by using Laplace transform techniques.

(10 marks)

Or

(b) (i) Find (1) $L^{-1} \left[\frac{1}{s(s-a)} \right]$; and (2) $L^{-1} \left[\log \frac{1-s^2}{s^2} \right]$.

(5 marks)

(ii) Solve the system of equations $\frac{dy}{dt} + ay = x$; $\frac{dx}{dt} + ax = y$ with $x(0) = 0$; $y(0) = 1$.

(10 marks)

4. (a) (i) Find the directional derivative of $\phi = x^2yz + 4xz^2$ at $(1, -2, -1)$ in the direction of $2i - j - 2k$.

(7 marks)

(ii) Prove that $\nabla \cdot (\vec{u} \times \vec{v}) = \vec{v} \cdot (\nabla \times \vec{u}) - \vec{u} \cdot (\nabla \times \vec{v})$.

(8 marks)

Or

(b) (i) Show that $\vec{F} = (6xy + z^3)\mathbf{i} + (3x^2 - z)\mathbf{j} + (3xz^2 - y)\mathbf{k}$ is irrotational and hence find its scalar potential.

(8 marks)

(ii) Prove that $\text{Curl} [f(r)\vec{r}] = 0$ if \vec{r} is the position vector and r is its magnitude.

(7 marks)

5. (a) (i) By changing the order of integration, evaluate $\int_0^1 \int_{x^2}^{2-x} xy \, dx \, dy$.

(6 marks)

- (ii) Evaluate $\iint_S \vec{F} \cdot \hat{n} \, ds$ if $\vec{F} = (x^2 - yz)\mathbf{i} + (y^2 - zx)\mathbf{j} + (z^2 - xy)\mathbf{k}$ and S is the surface of $0 \leq x \leq a$, $0 \leq y \leq b$, $0 \leq z \leq c$.

(9 marks)

Or

- (b) (i) Evaluate $\int_C \vec{F} \cdot d\vec{r}$ if $\vec{F} = (5xy - 6x^2)\mathbf{i} + (2y - 4x)\mathbf{j}$ and C is the curve in the xy -plane, $y = x^3$ from $(1, 1)$ to $(2, 8)$.

- (ii) Verify Stoke's theorem for $\vec{F} = (x^2 + y^2 - 4)\mathbf{i} + 3xy\mathbf{j} + (2xz + z^2)\mathbf{k}$ where S is the surface of the hemisphere $x^2 + y^2 + z^2 = 16$, above the xy -plane and C is its boundary.

(8 marks)

