C 26937 (Pages : 2) Name.............................. Reg. No...............................

COMBINED FIRST AND SECOND SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, JUNE 2003

EN2 K 101. MATHEMATICS—I

(New Scheme)

Time : Three Hours Maximum : 100 Marks

Answer **all** *the questions.*

1. (a) Find the radius of curvature at the origin on the curve $x^3 + y^3 + 2x^2 - 4y + 3x = 0$.

   (b) Verify Euler's theorem on homogeneous function $u(x, y) = \sin\left(\frac{xy}{x+y}\right)$.

   (c) Test for convergence the series $\sum_1^\infty \frac{1}{\left(1+\frac{1}{n}\right)^{n^2}}$.

   (d) Find the nth differential coefficient of $x^3 \sin^3 x$.

   (e) Find the rank of the matrix

$$A = \begin{pmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 16 & 4 & 12 & 15 \end{pmatrix}.$$

   (f) By Gauss elimination method, solve the equations

$$x + y + z = 9,\ 2x + 5y + 7z = 52,\ 2x + y - z = 0.$$

   (g) Find the Fourier series for $f(x) = x \sin x$ in $(-\pi, \pi)$.

   (h) Find the sine series for $f(x)$ in $(0, L)$, if

$$f(x) = x(L - x).$$

(8 × 5 = 40 marks)

2. (a) (i) Find the evaluate of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. (8 marks)

   (ii) If $z = f(x, y)$ and $x = r\cos\theta,\ y = r\sin\theta$, prove that

$$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2.$$

(7 marks)

Or

(b) (i) Find the maxima and minima of $x^2 - xy + y^2 - 2x + y$. (7 marks)

(ii) Find $\dfrac{\partial(x, y, z)}{\partial(r, \theta, \phi)}$, if $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$. (8 marks)

3. (a) (i) Test for convergence of the series $\dfrac{1\cdot 2}{3\cdot 4\cdot 5} + \dfrac{2\cdot 3}{4\cdot 5\cdot 6} + \dfrac{3\cdot 4}{5\cdot 6\cdot 7} + \ldots$ (7 marks)

(ii) Test for convergence of the series $\dfrac{1}{1+2} + \dfrac{2}{1+2^2} + \dfrac{3}{1+2^3} + \ldots$ (8 marks)

Or

(b) (i) If $y = \left(x + \sqrt{1+x^2}\right)^m$, prove that

$$(1+x^2)\, y_{n+2} + (2n+1)\, xy_{n+1} + (n^2 - m^2)\, y_{n=0}.$$

(8 marks)

(ii) By using Maclaurin's series expand $\cos x \sinh x$ in powers of x. (7 marks)

4. (a) (i) Show that the equations $2x + 3y + z = 9$, $x + 2y + 3z = 6$, $3x + y + 2z = 8$ are consistant. (7 marks)

(ii) Verify Cayley-Hamilton theorem for the matrix $A = \begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$. (8 marks)

Or

(b) (i) Show that the quadratic form $5x_1^2 + 26x_2^2 + 10x_3^2 + 6x_1x_2 + 4x_2x_3 + 14x_3x_1$ is positive semi-definite. (9 marks)

(ii) Diagonalize the matrix $A = \begin{pmatrix} 10 & 3 \\ 4 & 6 \end{pmatrix}$. (6 marks)

5. (a) (i) Obtain the Fourier series of $f(x) = x(2\pi - x)$ in $(0, 2\pi)$. (8 marks)

(ii) Find the Fourier series of $f(x) = x - x^2$ in $(-1, 1)$. (7 marks)

Or

(b) (i) By using cosine series for $y = x$ in $(0, \pi)$, show that $\dfrac{\pi^4}{96} = 1 + \dfrac{1}{3^4} + \dfrac{1}{5^4} + \dfrac{1}{7^4} + \ldots$ (8 marks)

(ii) Find the first two harmonics of the Fourier series for the following data :

x	:	0	$\pi/3$	$\dfrac{2\pi}{3}$	π	$\dfrac{4\pi}{3}$	$\dfrac{5\pi}{3}$	2π
y	:	1	1.4	1.9	1.7	1.5	1.2	1.0

(7 marks)