C 37067 (Pages : 3)

Name.........................

Reg. No.........................

FOURTH SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, JUNE 2004

CS 2K/IT 2K 401/PTCS 2K 401—ENGINEERING MATHEMATICS—IV

Time : Three Hours Maximum : 100 Marks

Answer **all** *questions.*

1. (a) Find the Fourier transform of $f(x)$ given by $f(x) = \begin{cases} 1 & \text{for } |x| < a \\ 0 & \text{for } |x| > a > 0 \end{cases}$.

   (b) Obtain the Fourier cosine transform of e^{-x^2}.

   (c) For what value k, the following function represents a probability mass function $f(x) = k\,3^{-x}$; $x = 1, 2, 3, \ldots$ Find its mean value.

   (d) An unbiased coin is tossed for 8 times. What is the probability of getting :

   (i) at least 3 heads.

   (ii) exactly 5 heads.

   (e) If the joint density function of the random variables X and Y is :

   $$f(x, y) = \begin{cases} 15xy\, e^{-5x-3y} & ; \ x > 0, y > 0 \\ 0 & ; \ \text{otherwise} \end{cases}$$

   then prove that X and Y are independent random variables.

   (f) The joint density of X and Y is given by

   $$f(x, y) = \begin{cases} 6xy(2 - x - y) & 0 < x < 1, \quad 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

   Obtain the conditional densities $f_{x|y}(x|y)$ and $f_{y|x}(y|x)$.

   (g) The one-step transition probability matrix of a 2-state Markov chain is given by $P = \begin{bmatrix} 0.5 & 0.5 \\ 0.3 & 0.7 \end{bmatrix}$ find $P^{(3)}$ and $P^{(4)}$.

   (h) Prove that the inter arrival times of Poisson process follow exponential distribution.

   (8 × 5 = 40 marks)

2. (a) (i) Find the Fourier transform of $f(x) = \begin{cases} 1 - |x| & \text{for } |x| < 1 \\ 0 & \text{otherwise} \end{cases}$. (8 marks)

   (ii) Obtain the Fourier cosine transform of $f(x) = \begin{cases} \cos x & \text{for } 0 < x < \pi \\ 0 & \text{otherwise} \end{cases}$. (7 marks)

Or

Turn over

(b) (i) Find $f(x)$, if its Fourier sine transform $F_s[f(x)] = \frac{w}{w^2+1}$. (8 marks)

(ii) Solve for $f(x)$ from the integral equation $\int_0^\infty f(x)\cos ax\,dx = e^{-a}$. (7 marks)

3. (a) (i) Find the distribution function of the random variable X whose probability density is given by :

$$f(x) = \begin{cases} \frac{x^2}{18} & 0 < x < 3 \\ \frac{(6-x)^2}{18} & 3 < x < 6 \\ 0 & \text{elsewhere} \end{cases}$$

Also find its mean.

(8 marks)

(ii) Determine the moment generating function of the random variable X whose density function $f(x) = \frac{1}{10}e^{-5|x|}$ $-\infty < x < \infty$ and hence its mean E (X).

(7 marks)

Or

(b) (i) The number of monthly breakdowns of a computer is a random variable having a Poisson distribution with $\lambda = 2.5$. Find the probability that this computer will function for a month :

(1) without a breakdown.

(2) not more that 3 breakdowns.

(3) exactly one breakdown.

(8 marks)

(ii) A balanced coin is tossed for 100 times using normal approximation determine the probability of getting :

(1) not more than 45 heads.

(2) more than 40 but less than 70 heads.

(7 marks)

4. (a) (i) If X and Y are independent random variables both uniformly distributed in (0,1), find the probability density of X + Y.

(8 marks)

(ii) The number of driving licenses issued in a certain city during a month is a random variable with mean 124 and standard deviation 7.5. Using Chebyshev's inequality with what probability we can assert that between 64 and 184 licences will be issued during a forth coming month ?

(7 marks)

Or

(b) (i) The joint density function of the random variable X and Y is given by $f(x,y)=\begin{cases} x+y & 0\le x\le 1 \\ & 0\le y\le 1 \\ 0 & \text{otherwise}\end{cases}$. Find the covariance between X and Y.

(8 marks)

(ii) The point probability mass function of X and Y is given by $P[X=i, Y=j]=e^{-(a+bi)}\dfrac{(bi)^j}{j!}\dfrac{a^i}{i!}\ i\ge 0, j\ge 0$. Find the conditional distribution of Y given that $X=i$.

(7 marks)

5\. (a) (i) A three state Markov chain with transition probability matrix is given by

$$P=\begin{bmatrix}0.4 & 0.1 & 0.5\\ 0.4 & 0.3 & 0.3\\ 0.3 & 0.5 & 0.2\end{bmatrix}.$$

Find the limiting probabilities.

(8 marks)

(ii) The repair time required for a certain type of machine is exponential distributed with a mean of 5 hours. What is the probability that a repair time exceeds 7 hours ? What is the probability that the repair is over within 8 hours given that already the repair takes 3 hours ?

(7 marks)

Or

(b) (i) Show that S_n, the arrival time of n th event in Poisson process follows the density function

$$f_{s_n}(t)=\lambda e^{-\lambda t}\frac{(\lambda t)^{n-1}}{(n-1)!}.$$

(8 marks)

(ii) Motor cars pass a point on a highway at a Poisson rate of 5 per minute in which 5 per cent of the motor cars are vans. What is the probability that atleast one van passes by during an hour ? If 25 motor cars have passed in one hour, what is the probability that 5 of them are vans ?

(7 marks)

[4 × 15 = 60 marks]

(b) (i) Find $f(x)$, if its Fourier sine transform $F_s[f(x)] = \frac{w}{w^2+1}$. (8 marks)

(ii) Solve for $f(x)$ from the integral equation $\int_0^\infty f(x)\cos ax\,dx = e^{-a}$. (7 marks)

3. (a) (i) Find the distribution function of the random variable X whose probability density is given by :

$$f(x) = \begin{cases} \frac{x^2}{18} & 0 < x < 3 \\ \frac{(6-x)^2}{18} & 3 < x < 6 \\ 0 & \text{elsewhere} \end{cases}$$

Also find its mean.

(8 marks)

(ii) Determine the moment generating function of the random variable X whose density function $f(x) = \frac{1}{10}e^{-5|x|} - \infty < x < \infty$ and hence its mean E (X).

(7 marks)

Or

(b) (i) The number of monthly breakdowns of a computer is a random variable having a Poisson distribution with $\lambda = 2.5$. Find the probability that this computer will function for a month :

(1) without a breakdown.

(2) not more that 3 breakdowns.

(3) exactly one breakdown.

(8 marks)

(ii) A balanced coin is tossed for 100 times using normal approximation determine the probability of getting :

(1) not more than 45 heads.

(2) more than 40 but less than 70 heads.

(7 marks)

4. (a) (i) If X and Y are independent random variables both uniformly distributed in (0,1), find the probability density of X + Y.

(8 marks)