C 37045 (Pages : 2) Name..............................

Reg. No..............................

FOURTH SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, JUNE 2004

CH 2K 401. ENGINEERING MATHEMATICS—IV

(Common to AI/CE/EE/IC/ME/PM/EC/PE)

Time : Three Hours Maximum : 100 Marks

Answer **all** *questions.*

1. (a) Show that the function $u = 3x^2y - y^3$ is harmonic and find a corresponding analytic function $f(z) = u(x, y) + iv(x, y)$.

   (b) Find the map of the circle $|z| = k$ by the transformation $w = z + 4 + 13i$.

   (c) Using Cauchy's integral formula evaluate

   $$\int_C \frac{z}{(z-1)(z-2)}\, dz, \text{ where C is } |z-2| = \frac{1}{2}.$$

   (d) Expand $f(z) = \sin z$ in a Taylor's series about $z = \frac{\pi}{4}$.

   (e) Show that $J_0'' = \frac{1}{2}(J_2 - J_0)$.

   (f) Expand $1 + x - x^2$ in a series of Legendre polynomials.

   (g) Prove that if f and g are arbitrary twice differentiable functions, then $y = f(x - at) + g(x + at)$ is a solution of the one-dimensional wave equation.

   (h) Classify the following partial differential equation :

   $$u_{xx} + 4u_{xy} + \left(x^2 + 4y^2\right) u_{yy} = \sin(x + y).$$

   (8 × 5 = 40 marks)

2. (a) (i) If $u(x, y)$ and $v(x, y)$ are harmonic functions in a region R, prove that the function $\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) + i\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$ is an analytic function of $z = x + iy$.

   (ii) Find the bilinear transformation which maps $0, 1, \infty$ onto $i, -1, -i$ respectively.

   Or

   (b) (i) Converting into polar form show that when z is not zero, $\log z$ satisfies the Cauchy-Riemann equations.

   (ii) Show that the transformation $w = z + \frac{1}{z}$ maps the upper half of the interior of the unit circle $|z| = 1$ onto the lower half of the w-plane.

   (15 marks)

Turn over

3. (a) (i) If $f(a) = \int_C \frac{4z^2 + z + 5}{z - a} dz$, where C is the ellipse $9x^2 + 4y^2 = 36$, find the values of $f(1), f(i)$, $f'(-1)$ and $f''(-i)$.

(ii) State and prove Cauchy's residue theorem.

Or

(b) (i) Find Laurent's series in the region $2 < |z| < 3$ if $f(z) = \frac{z^2 - 1}{(z+2)(z+3)}$.

(ii) Using Contour integration, evaluate $\int_0^{2\pi} \frac{\cos 2\theta \, d\theta}{5 + 4\cos\theta}$.

(15 marks)

4. (a) (i) State and prove orthogonality property of Bessel functions.

(ii) Solve in series the differential equation :

$$2x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + \left(x^2 + 1\right) y = 0.$$

Or

(b) (i) State and prove Rodrigue's formula for Legendre polynomials.

(ii) Prove that $\int_{-1}^{1} x P_n(x) P_{n-1}(x) \, dx = \frac{2n}{4n^2 - 1}$.

(15 marks)

5. (a) (i) Derive one-dimensional wave equation.

(ii) A string of length l is initially at rest in equilibrium position and each of its points is given the velocity

$$\left(\frac{\partial y}{\partial t}\right)_{t=0} = b \sin^3\left(\frac{\pi x}{l}\right).$$

Find the displacement $y(x, t)$.

Or

(b) (i) Explain the method of separation of variables for solving one-dimensional heat equation.

(ii) A bar 10 cm. long, with insulated sides, has its ends A and B maintained at temperature 50° C. and 100° C. respectively, until steady-state conditions prevail. The temperature at A is suddenly raised to 90° C. and at the same time that at B is lowered to 60° C. Find the temperature distribution in the bar at time t.

(15 marks)

[4 × 15 = 60 marks]