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Name...

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FIFTH SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, DECEMBER 2004

CH 2K 506 A. NUMERICAL ANALYSIS

(Common to AI/CE/EC/EE/IC/ME/PM/PTME 2K 505 A/PTEE 2K 504 D)

Time : Three Hours

Maximum : 100 Marks

Answer **all** *questions.*

1. (a) Explain Newton-Raphson method of solving an algebraic equation.

   (b) Using Bairstow's method obtain the quadratic factor of the equation

   $$x^4 - 3x^3 + 20x^2 + 44x + 54 = 0, \text{ with } (p, q) = (2, 2).$$

   Perform two iterations.

   (c) Using Gauss-Jacobi method, solve the system of equations

   $$10x - 5y - 2z = 3,\ 4x - 10y + 3z = -3,\ x + 6y + 10z = -3$$

   upto second iteration.

   (d) By Gauss elimination method solve $2x + y = 3$, $7x - 3y = 4$.

   (e) Given that $f(-1) = -2, f(0) = -1, f(2) = 1, f(3) = 4$, fit a polynomial of third degree.

   (f) Find $y'(0)$ from the following table :—

x	:	0	1	2	3	4	5
y	:	4	8	15	7	6	2

   (g) Using Taylor series method, solve $y' = xy + y^2, y(0) = 1$ at $x = 0{\cdot}1$.

   (h) Find $y(0{\cdot}2)$ given $y' = 3x + \frac{1}{2}y, y(0) = 1$ by using Runge-Kutta Fourth order method.

   (8 × 5 = 40 marks)

2. (a) (i) Find the root of the equation $x^3 - 2x - 5 = 0$ lying between 2 and 3, correct to 3 places of decimals, using method of false position.

   (ii) Apply Graeffe's root squaring method to find the root of $x^3 - 2x + 2 = 0$ correct to two decimal places.

   Or

   (b) (i) Compute the positive root of the equation $x - \cos x = 0$, correct to 2 places of decimals using the bisection method.

   (ii) Use Muller's method to find a root of the equation $x^3 - x - 2 = 0$, $x_1 = 1$, $x_2 = 1{\cdot}2$ and $x_3 = 1{\cdot}4$.

3. (a) (i) Solve the following system of equations by Gauss-Seidel iteration method $2x + y + z = 4$, $x + 2y + z = 4$, $x + y + 2z = 4$.

   (ii) Use Newton-Raphson method to solve the equations $x = x^2 + y^2, y = x^2 - y^2$. Correct to two decimals, starting with the approximation (0·8, 0·4).

   Or

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(b) (i) Solve by Crout's method the equations $10x + y + 2z = 13$, $3x + 10y + z = 14$, $2x + 3y + 10z = 15$.

(ii) Find by power method the largest eigenvalue and the corresponding eigenvector of the matrix :

$$\begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}.$$

4. (a) (i) Using Newton's divided difference table, find $f(x)$ which takes the values

$1, 4, 40, 85$ as $x = 0, 1, 3, 4$.

(ii) By dividing the range into ten equal parts, evaluate $\int_0^{\pi} \sin x \, dx$ by Trapezoidal rule and Simpson's $\frac{1}{3}$ rule. Verify your answer with integration.

Or

(b) (i) Interpolate y at $x = 5$ given :

x	:	1	2	3	4	7
y	:	2	4	8	16	128

(ii) Calculate $\Delta^4 u_6$ if $u_6 = 2$, $u_7 = -6$, $u_8 = 8$, $u_9 = 9$ and $u_{10} = 17$.

5. (a) (i) Use Euler's method and its modified form to obtain $y(0{\cdot}2)$ and $y(0{\cdot}4)$ correct to three decimal places given that $y' = y - x^2$, $y(0) = 1$.

(ii) Solve the hyperbolic partial differential equation (vibration of strings) for one half period of oscillation taking $h = 1$.

$$u_{tt} = 25u_{xx},\ u(0, t) = u(5, t) = 0\ ;\ u_t(x, 0) = 0$$

$$u(x, 0) = \begin{cases} 2x & \text{for } 0 \le x \le 2{\cdot}5 \\ 10 - 2x & \text{for } 2{\cdot}5 \le x \le 5. \end{cases}$$

Or

(b) (i) Using Runge-Kutta algorithm of second order solve

$$\frac{dy}{dx} + y = 0,\ \ y(0) = 1 \ \text{ for } y(0{\cdot}1) \text{ and } y(0{\cdot}2).$$

(ii) Solve $y' = x^2 + y^2 - 2$, using Milne's method for $x = 0{\cdot}3$ given that $y = 1$ at $x = 0$. Compute $y(-0{\cdot}1)$, $y(0{\cdot}1)$, $y(0{\cdot}2)$ using Runge-Kutta method of fourth order.

(4 × 15 = 60 marks)