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(2 Pages)

Name.....................................

Reg. No.................................

THIRD SEMESTER B.Tech. (ENGINEERING) DEGREE EXAMINATION DECEMBER 2004

(New Scheme)

RTCS 2K 303/IT/CS2K 303—DISCRETE COMPUTATIONAL STRUCTURES

Time : Three Hours — Maximum : 100 Marks

Answer **all** *the questions.*

1. (a) Obtain the disjunctive normal form of $(P \rightarrow Q) \vee \left(R \rightleftarrows P\right) \wedge (Q \vee R)$.

   (b) Show that R is a valid inference from the premises $P \rightarrow Q$, $Q \rightarrow R$ and P.

   (c) In a Boolean algebra, show that (i) $a * a = a$; (ii) $a \oplus 1 = 1$.

   (d) Give a relation which is both a partial ordering relation and an equivalence relation on a set. Also draw the Hasse diagram for the relation.

   (e) Show that the set of all polynomials in x under the operation of addition is a group.

   (f) Prove that a subset $S \neq \phi$ of G is a subgroup of $(G, *)$ iff for any pair of elements $a, b \in S$, $a * b^{-1} \in S$.

   (g) Prove that any field is an integral domain.

   (h) If R is a ring and $a \in R$, let $r(a) = \{x \in R / ax = 0\}$. Prove that $r(a)$ is a right ideal of R.

   (8 × 5 = 40 marks)

2. (a) (i) Show that $(x)(P(x) \vee Q(x)) \Rightarrow (x) P(x) \vee (\exists x) Q(x)$. (8 marks)

   (ii) Show that $(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$ without using truth table. (7 marks)

   Or

   (b) (i) Show that $((P \vee Q) \wedge \neg(\neg P \wedge (\neg Q \vee \neg R))) \vee (\neg P \wedge \neg Q) \vee (\neg P \wedge \neg R)$ is a tautology. (7 marks)

   (ii) Show that $P \vee Q, \quad Q \vee R, R \rightarrow S \Rightarrow P \rightarrow S$. (8 marks)

3. (a) (i) Show that the set Q of rational numbers is denumerable. (8 marks)

   (ii) Let $f : R \rightarrow R$ and $g : R \rightarrow R$ where R is the set of real numbers. Find fog and gof, where $f(x) = x^2 - 2$ and $g(x) = x + 4$. State whether these functions are injective, surjective and bijective. (7 marks)

   Or

Turn over

(b) (i) Prove that relation R given by $R = \{(x, y)/x - y \text{ is divisible by } m\}$ over the set of positive integers is an equivalence relation. Show also that if $x_1 R y_1$ and $x_2 R y_2$, then $(x + x_2) R (y_1 + y_2)$.

(8 marks)

(ii) Prove that $(A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D)$. (7 marks)

4\. (a) (i) Prove that a code can detect all combinations of K or fewer errors if and only if the minimum distance between any *two* code words is at least K + 1.

(8 marks)

(ii) Prove that every group of order 4 is abelian. (7 marks)

Or

(b) (i) Prove that an infinite cyclic group is isomorphic to the group $(z, +)$. (8 marks)

(ii) Prove that every subgroup of an abelian group is normal. Is the converse true ? Justif your answer ?

(7 marks)

5\. (a) (i) If L is an algebraic extension of K and if K is an algebraic extension of F, then prove that L is an algebraic extension of F.

(8 marks)

(ii) Prove that it is impossible to trisect 60° by straitedge and compass alone. (7 marks)

Or

(b) (i) Given two polynomials $f(x)$ and $g(x)$ over a field F, show that there exists two polynomials $t(x)$ and $r(x)$ in $F[x]$ such that $f(x) = t(x) g(x) + r(x)$ where $r(x) = 0$ or deg. $r(x) <$ deg. $g(x)$.

(8 marks)

(ii) Prove that $x^3 - 9$ is reducible over the integers mod 11. (7 marks)

[4 × 15 = 60 marks]