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(Pages : 2)

Name.........................

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THIRD SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, DECEMBER 2004

CS/IT 2K/PTCS 2K 301. ENGINEERING MATHEMATICS—III

Time : Three Hours

Maximum : 100 Marks

Answer **all** *questions.*

1. (a) Show that the vectors (1, 1, 1), (1, 0, – 1) and (1, – 2, 1) are mutually orthogonal.

   (b) A linear transformation T on R^3 is given by $T(x, y, z) = (-z, x, x+z)$. What is the image of (– 3, 10, 5) under T ?

   (c) If A is a symmetric matrix, show that

   $$y^T A x = x^T A y.$$

   (d) Show that the characteristic value problem :

   $$2x_1 - 2x_2 = \lambda x_1$$
   $$4x_1 - 2x_2 = \lambda x_2$$

   has no real nontrivial solutions for any value of λ.

   (e) In a complex electric field, the potential is $\omega = \phi + i\psi$, $\psi = x^3 - 3xy^2$. Find ϕ.

   (f) Find the map of the circle $|z| = k$ by the transformation $w = (1 + 2i)z + 1 + i$.

   (g) Find the Laurent expansion of $f(z) = \dfrac{1}{z(z+1)^2}$ in $0 < |z| < 1$.

   (h) Calculate the residues of $f(z) = \dfrac{z}{z^2 + 2z + 5}$ at its isolated singular points.

(8 × 5 = 40 marks)

2. (a) The vectors (4, – 1, 3) and (2, 1, – 1) span a subspace U and the vectors (3, 3, 4) and (5, 4, 11) span a subspace W, of R^3. Are U and W the same ?

   (b) Find a basis, and the dimension, of the vector space V spanned by the vectors (3, – 1, 2), (7, 4, – 2), (8, 1, 5).

   Or

   (c) Using the Gram-Schmidt orthogonalisation process, find an orthonormal basis for the subspace of R^3 spanned by the vectors (2, – 14, 5), (6, – 12, 9).

   (d) Let T_1, T_2 and T_3 be linear transformations and let C be an arbitrary scalar. Show that :

   (i) $T_1(T_2T_3) = (T_1T_2)T_3$.

   (ii) $(CT_1)T_2 = T_1(CT_2) = C(T_1T_2)$.

Turn over

3. (a) Show that, for every matrix A, the row rank, the column rank, and the determinant rank are all equal.

(b) Reduce the quadratic form $3x^2 + 5y^2 + 3z^2 - 2xy - 2yz + 2zx$ to canonical form.

Or

(c) Reduce the matrix $\begin{pmatrix} -1 & 1 & 2 \\ 0 & -2 & 1 \\ 0 & 0 & -3 \end{pmatrix}$ to a diagonal form.

(d) Solve the following matrix equation :—

$$X^2 + 6X + 9I = \begin{bmatrix} -5 & 9 \\ -6 & 10 \end{bmatrix}.$$

4. (a) If $f(z)$ is a regular function of z, prove that $\left(\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}\right) |f(z)|^2 = 4\,|f'(z)|^2$.

(b) Find the bilinear transformation that maps $z = 1$, $z = i$ and $z = -1$ onto the points $w = 0$, $w = 1$ and $w = \infty$ of the w-plane.

Or

(c) Derive Cauchy-Riemann equations in Cartesian coordinates.

(d) Show that the transformation $w = \dfrac{z}{z-1}$ maps the upper half of the z-plane onto the upper half of the w-plane.

5. (a) Using Cauchy's integral formula for derivatives, evaluate

$$\int_C \frac{z^3 - z}{(z+2)^3}\, dz,\ c : 16x^2 + 25y^2 = 400.$$

(b) Evaluate $\displaystyle\int_0^{2\pi} \frac{d\theta}{5 - 4\sin\theta}$ by contour integration.

Or

(c) Obtain Taylor series expansion for the function $f(z) = \dfrac{z^3 + 2z^2}{z^2 + 2z + 3}$ in $|z| < \dfrac{1}{2}$.

(d) State and prove Cauchy's residue theorem.

(4 × 15 = 60 marks)