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THIRD SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, DECEMBER 2004

CH 2K 301. ENGINEERING MATHEMATICS—III

(Common to AI/CE/EE/IC/ME/PM/EC/PE/PT/PTCE/PTEE/PTCH/PTME 2K 301)

Time : Three Hours

Maximum : 100 Marks

Answer **all** *questions.*

1. (a) Determine whether the set of vectors (2, 3, – 1), (3, 2, 2), (4, 4, – 1) in R^3 is linearly dependent or linearly independent.

   (b) A linear transformation T on R^3 is given by $T(x, y, z) = (-z, x, x + z)$. Find the kernel of T and interpret it geometrically.

   (c) If A is an $m \times 1$ matrix and B is a $1 \times n$ matrix, neither of which is a zero matrix, what is the rank of AB ?

   (d) Find the matrix of the quadratic form :

$$3x_1^2 + 3x_2^2 + 6x_3^2 - 2x_1x_2 - 4x_1x_3.$$

   (e) A certain screw making machine produces on average of 2 defective screws out of 100, and packs them in boxes of 500. Find the probability that a box contains 15 defective screws.

   (f) Show that for the uniform distribution $f(x) = 1,\ 0 \le x \le 1$, mean $= \frac{1}{2}$.

   (g) Two samples of sizes 9 and 8 give the sum of the squares of deviations from their respective means equal to 160 and 91 respectively. Can they be regarded as drawn from the same normal population ?

   (h) Discuss the Chi-square test of goodness of fit. What cautions are necessary while applying the test ?

(8 × 5 = 40 marks)

2. (a) Verify that the vectors $u = (3, -2, 1)$ and $v = (1, 2, 1)$ are orthogonal and find a vector w such that u, v and w are mutually orthogonal.

   (b) Prove that the Schwarz inequality is equivalent to the triangle inequality :

$$\|u + v\| \le \|u\| + \|v\|.$$

Or

   (c) (i) Does the transformation $T(x, y, z) = (x + 2y + z, x + 3y - 2z)$ map R^3 onto R^2 ?

   (ii) Is T an isomorphism ? Explain.

   (d) Show that if U is a subspace of a finite dimensional vector space V, then dim U ≤ dim V. If dim U = dim V, show that U = V.

(15 marks)

Turn over

3. (a) Determine the rank of the matrix

$$\begin{vmatrix} 8(1-\lambda) & -2 & 0 \\ -2 & 3-2\lambda & -1 \\ 0 & -1 & 2(1-\lambda) \end{vmatrix}$$

as a function of λ.

(b) Prove that if a 2×2 matrix has characteristic vectors which are orthogonal, it is symmetric.

Or

(c) Find a similarity transformation which will reduce the matrix

$$\begin{pmatrix} 5 & -2 & -1 \\ -1 & 4 & -1 \\ 1 & -2 & 3 \end{pmatrix}$$

to a diagonal form.

(d) Verify Cayley-Hamilton theorem for the matrix

$$\begin{pmatrix} -1 & -2 \\ 3 & 4 \end{pmatrix}.$$

(15 marks)

4. (a) In a large consignment of electric bulbs, 10 % are defective. A random sample of 20 is taken for inspection. Find the probability that (i) all are good bulbs ; (ii) atmost there are 3 defective bulbs ; and (iii) there are exactly 3 defective bulbs.

(b) The probability density $p(x)$ of a continuous random variable is given by

$$p(x) = y_0 e^{-|x|}, -\infty < x < \infty.$$

Prove that $y_0 = \frac{1}{2}$. Find the mean and variance of the distribution.

Or

(c) Show that the mode of the geometric distribution $P(x) = \left(\frac{1}{2}\right)^x$, $x = 1, 2, 3, \ldots$ is unity.

(d) Find the moment generating function of the Gamma distribution

$$f(x) = \frac{1}{\Gamma\left(\frac{1}{4}\right)} e^{-x} x^{-3/4}, x \geq 0,$$

at the origin.

(15 marks)

5. (a) In a sample of 600 men from a certain city, 450 are found smokers. In another sample of 900 men from another city, 450 are smokers. Do the data indicate that the cities are significantly different with respect to the habit of smoking among men ?

(b) A sample of 10 measurements of the diameter of a sphere gave a mean of 12 cm. and a standard deviation 0·15 cm. Find 95 % confidence limits for the actual diameter.

Or

(c) A random sample of 10 boys had the following I.Q. :—

70, 120, 110, 101, 88, 83, 95, 98, 107, 100.

Do these data support the assumption of a population mean I.Q. of 100 (at 5 % level of significance) ?

(d) Explain the following terms :

(i) Null hypothesis and alternative hypothesis.

(ii) Type I and Type II errors.

(15 marks)

[4 × 15 = 60 marks]