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C 6252 (Pages : 3)

Name.................................

Reg. No.................................

SIXTH SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, JUNE 2005

CS 2K 603. GRAPH THEORY AND COMBINATORICS

(New Scheme)

Time : Three Hours — Maximum : 100 Marks

Answer **all** *questions.*

Part A

1. (a) Define adjacency matrix and incidence matrix with an example for each.

   (b) Show that for every planar graph $q \leq 3p - 6$, where p, q are number vertices and edges of the graph.

   (c) Draw the tree structure of $(a + 5) \times [(3b + c)/(d + 2)]$ and write the prefix notation.

   (d) Prove that every tree has $p - 1$ edges, where p is the number of vertices of the tree.

   (e) In how many ways can a committee of 5 teachers and 4 students be chosen from 9 teachers and 15 students ? Also find the number of ways if one particular teacher refuses to serve if one particular student is on the committee ?

   (f) Using $\sum_{k=0}^{n-1} C(k + 2, 2) = C(n + 2, 3)$ obtain the prove of

   $$1{\cdot}2 + 2{\cdot}3 + 3{\cdot}4 + \cdots + n(n - 1) = \frac{1}{3} n(n + 1)(2 + n).$$

   (g) How many different license plates are there that involve 1, 2 or 3 letters followed by 4 digits ?

   (h) Find the generating function of $\sum_{n=0}^{\infty} n^3 a^n X^n$.

(8 × 5 = 40 marks)

Part B

2. (a) (i) Discuss the platonic bodies. (8 marks)

   (ii) Discuss the chinese problem and its solution. (7 marks)

   Or

   (b) (i) State Kurtowski's theorem and its applications. (8 marks)

   (ii) Discuss the map colouring with the existing results. (7 marks)

Turn over

12. (a) Compute (i) A (1, 1) ; (ii) A (1, 2) ; and (iii) A (2, 1), where $A : N^2 \to N$ is defined by

$$A(0, y) = y + 1,\ A(x+1, 0) = A(x, 1),$$
$$A(x+1, y+1) = A(x+1, y).$$

(b) Solve the recurrence relation :

$a_r = a_{r-1} + f(r)$ for $r \geq 1$ by substitution.

(8 + 7 = 15 marks)

Or

(c) Solve the recurrence relation

$$a_r - 7a_{r-1} + 12a_{r-2} = 0.$$

(d) Find the particular solution of

$$a_r + 5a_2 r^m + 6a_{r-2} = 3r_2.$$

(8 + 7 = 15 marks)

[4 × 15 = 60 marks]

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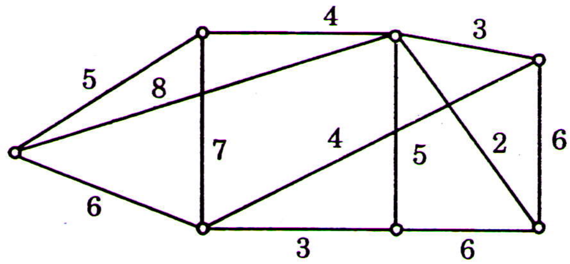
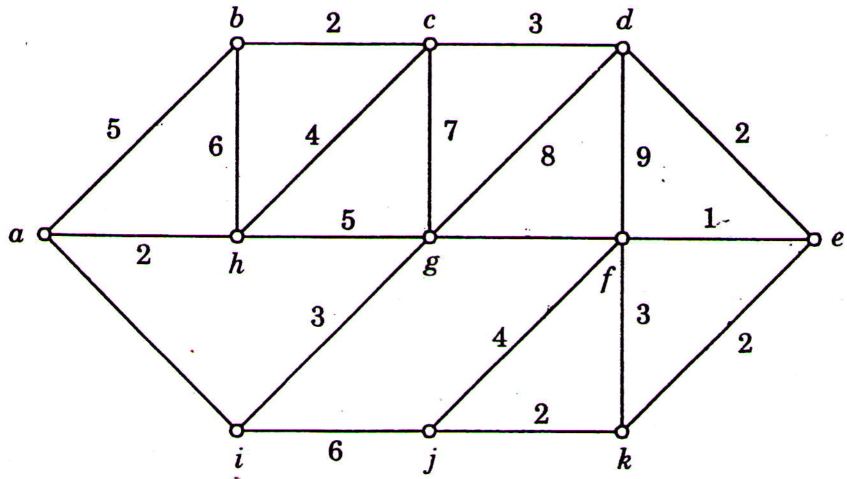
3. (a) Write Kruskal's algorithm and find the minimal spanning tree of the graph :



Or

(b) Write Dijkstra's algorithm and find the distances of the other vertices of the following graph from the vertex a :—



(15 marks)

4. (a) (i) In how many ways can the letters $\{5 \cdot a, 4 \cdot b, 3 \cdot c\}$ be arranged so that all the letters of the same kind are not in a single block.

(7 marks)

(ii) A sample of 5 balls is to be selected from distinguishable 15 red balls and 5 white balls. Then find (1) How many samples of 5 balls are there ? (2) How many samples contain all red balls ? (3) How many samples contain 3 red balls and 2 white balls ? (4) How many samples contain at least 4 red balls ?

(8 marks)

Or

(b) (i) Count the number of integral solutions to $x_1 + x_2 + x_3 = 20$, where $2 \le x_1 \le 5$, $4 \le x_2 \le 7$ and $-2 \le x_3 \le 9$.

(8 marks)

(ii) Find how different license plates are there that involve :

(1) 1, 2 or 3 letters followed by 4 digits.

(2) 1, 2 or 3 letters followed by 1, 2, 3 or 4 digits.

(7 marks)

5. (a) (i) Find the generating function for the sequence $A = \{a_n\}_{r=0}^{\infty}$, where $a_r = \begin{cases} 1 & \text{if} \quad 0 \le r \le 2 \\ 3 & \text{if} \quad 3 \le r \le 5 \\ 0 & \text{if} \quad r \le 6. \end{cases}$

(7 marks)

(ii) Find the complete solution of $a_{n+1} + 2a_{n-1} = n + 3$. (8 marks)

Or

(b) (i) Solve recurrence relation $a_n - a_{n-1} = 2(n-1)$ for $n \ge 1$ and $a_0 = 2$. (8 marks)

(ii) Find the complete solution to $a_n - 10\, a_{n-1} + 25\, a_{n-2} = 2^n$ where $a_0 = 2/3$ and $a_1 = 3$.

(7 marks)

[4 × 15 = 60 marks]