C 5763 (2 Pages) Name..............................

Reg. No..............................

COMBINED FIRST AND SECOND SEMESTER B.Tech. (ENGINEERING) DEGREE EXAMINATION, JUNE 2005

EN2K 102—MATHEMATICS—II

(Common to all Branches)

Time : Three Hours Maximum : 100 Marks

Part A

Each question carries 5 marks.

I. (a) Solve $(D^2 + 5)\, y = \cos(5x)$.

(b) Solve $\dfrac{dy}{dx} = \dfrac{x^2 + xy + y^2}{x^2}$.

(c) Find $L(t^2 e^t \sin t)$.

(d) Find $L^{-1}\left(\dfrac{s+1}{\left(s^2 + 2s + 2\right)^2}\right)$.

(e) If $\vec{r}$ is the position vector of any point P (x, y, z), prove that $\text{grad}\, r^n = n r^{n-2} \vec{r}$.

(f) Show that $\text{div}\left(\dfrac{\vec{r}}{r}\right) = \dfrac{2}{r}$.

(g) Use Green's theorem to evaluate $\int_C x^2 y\, dx + y^3 dy$, where C is the closed path formed by $y = x$ and $y = x^3$ from (0, 0) to (1, 1).

(h) Evaluate $\int_C \vec{f} \cdot d\vec{r}$, where $\vec{f} = \left(x^2 + y^2\right)\vec{i} + \left(x^2 - y^2\right)\vec{j}$ and C is the curve $y = x^2$ joining (0, 0) to (1, 1).

(8 × 5 = 40 marks)

Part B

II. (a) Solve $\dfrac{dy}{dx} = \dfrac{2x + 3y + 5}{3x + 2y + 7}$. (7 marks)

(b) Solve $x^2 y'' - x y' + 4y = \cos(\log(x)) + x \sin(\log(x))$. (8 marks)

Or

Turn over

(c) A plate is heated to 80 °C at time $t = 0$ and it is place in water, which is maintained at 30 °C. If at time $t = 3$ minutes, the temperature of the plate is 50 °C, find the time at which the temperature of the ball is 40 °C.

(8 marks)

(d) Solve $(D^2 - 6D + 9)y = 6e^{3x} + 7e^{-2x} - \log z + 3^x$. (7 marks)

III. (a) Solve $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} - 3y = \sin(t)$ given $y(0) = 0,\ y'(0) = 0$, when $t = 0$ using Laplace transform.

(15 marks)

Or

(b) Using Laplace transform solve $(D^2 + D)y = t^2 + 2t$, where $y(0) = 4$ and $y'(0) = -2$. (15 marks)

IV. (a) Find the angle between the surfaces $z = x^2 + y^2 - 3$ and $x^2 + y^2 + z^2 = 9$ at $(2, -1, 2)$. (8 marks)

(b) Prove that $\text{curl}(\bar{u} \times \bar{v}) = (\bar{v} \cdot \nabla)\bar{u} - (\bar{u} \cdot \nabla)\bar{v} + \bar{u}\,\text{div}\,\bar{v} - \bar{v}\,\text{div}\,\bar{u}$. (7 marks)

Or

(c) If $\bar{r}$ is the position vector of the point P (x, y, z), then prove (i) $\text{div}\,\bar{r} = 3$; (ii) $\text{curl}\,\bar{r} = \bar{0}$; (iii) $\nabla r^n = n r^{n-2}\bar{r}$ where $r = |\bar{r}|$.

(15 marks)

V. (a) Verify Gauss divergence theorem for $\vec{F} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ taken over the cube bounded by the planes $x = 0, x = 1 ; y = 0, y = 1 ; z = 0$ and $z = 1$.

(15 marks)

Or

(b) Verify Stoke's theorem for $\vec{f} = (x^2 - y^2)\vec{i} + 2xy\vec{j}$ in the rectangular region $x = 0, y = 0$; $x = a, y = b$.

(15 marks)

[4 × 15 = 60 marks]