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(2 Pages)

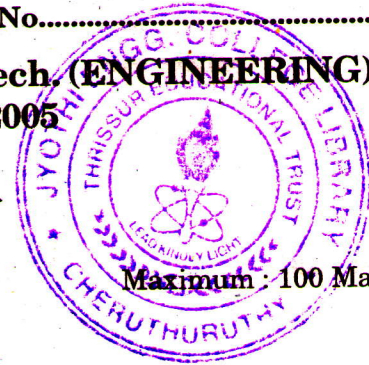
Name.....

Reg. No.....

**COMBINED FIRST AND SECOND SEMESTER B.Tech. (ENGINEERING)  
DEGREE EXAMINATION, JUNE 2005**

**EN2K 102—MATHEMATICS—II**

(Common to all Branches)



Time : Three Hours

Maximum : 100 Marks

**Part A**

Each question carries 5 marks.

I. (a) Solve  $(D^2 + 5)y = \cos(5x)$ .

(b) Solve  $\frac{dy}{dx} = \frac{x^2 + xy + y^2}{x^2}$ .

(c) Find  $L(t^2 e^t \sin t)$ .

(d) Find  $L^{-1}\left(\frac{s+1}{(s^2+2s+2)^2}\right)$ .

(e) If  $\vec{r}$  is the position vector of any point P (x, y, z), prove that  $\text{grad } r^n = n r^{n-2} \vec{r}$ .

(f) Show that  $\text{div}\left(\frac{\vec{r}}{r}\right) = \frac{2}{r}$ .

(g) Use Green's theorem to evaluate  $\int_C x^2 y dx + y^3 dy$ , where C is the closed path formed by  $y = x$  and  $y = x^3$  from (0, 0) to (1, 1).

(h) Evaluate  $\int_C \vec{f} \cdot d\vec{r}$ , where  $\vec{f} = (x^2 + y^2)\vec{i} + (x^2 - y^2)\vec{j}$  and C is the curve  $y = x^2$  joining (0, 0) to (1, 1).

(8 × 5 = 40 marks)

**Part B**

II. (a) Solve  $\frac{dy}{dx} = \frac{2x + 3y + 5}{3x + 2y + 7}$ . (7 marks)

(b) Solve  $x^2 y'' - x y' + 4y = \cos(\log(x)) + x \sin(\log(x))$ . (8 marks)

Or

Turn over

- (c) A plate is heated to  $80^\circ\text{C}$  at time  $t = 0$  and it is placed in water, which is maintained at  $30^\circ\text{C}$ . If at time  $t = 3$  minutes, the temperature of the plate is  $50^\circ\text{C}$ , find the time at which the temperature of the ball is  $40^\circ\text{C}$ .

(8 marks)

(d) Solve  $(D^2 - 6D + 9)y = 6e^{3x} + 7e^{-2x} - \log z + 3^x$ .

(7 marks)

- III. (a) Solve  $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} - 3y = \sin(t)$  given  $y(0) = 0$ ,  $y'(0) = 0$ , when  $t = 0$  using Laplace transform.

(15 marks)

Or

- (b) Using Laplace transform solve  $(D^2 + D)y = t^2 + 2t$ , where  $y(0) = 4$  and  $y'(0) = -2$ . (15 marks)

- IV. (a) Find the angle between the surfaces  $z = x^2 + y^2 - 3$  and  $x^2 + y^2 + z^2 = 9$  at  $(2, -1, 2)$ . (8 marks)

- (b) Prove that  $\text{curl}(\vec{u} \times \vec{v}) = (\vec{v} \cdot \nabla)\vec{u} - (\vec{u} \cdot \nabla)\vec{v} + \vec{u} \text{div} \vec{v} - \vec{v} \text{div} \vec{u}$ . (7 marks)

Or

- (c) If  $\vec{r}$  is the position vector of the point P  $(x, y, z)$ , then prove (i)  $\text{div} \vec{r} = 3$ ; (ii)  $\text{curl} \vec{r} = \vec{0}$ ; (iii)  $\nabla r^n = n r^{n-2} \vec{r}$  where  $r = |\vec{r}|$ .

(15 marks)

- V. (a) Verify Gauss divergence theorem for  $\vec{F} = x^2 \vec{i} + y^2 \vec{j} + z^2 \vec{k}$  taken over the cube bounded by the planes  $x = 0, x = 1; y = 0, y = 1; z = 0$  and  $z = 1$ .

(15 marks)

Or

- (b) Verify Stoke's theorem for  $\vec{f} = (x^2 - y^2) \vec{i} + 2xy \vec{j}$  in the rectangular region  $x = 0, y = 0; x = a, y = b$ .

(15 marks)

[4 × 15 = 60 marks]