C 5762 (2 Pages)

Name...

Reg. No...

COMBINED FIRST AND SECOND SEMESTER B.Tech. (ENGINEERING) DEGREE EXAMINATION, JUNE 2005

EN 2K 101—MATHEMATICS—I

(Common to all Branches)

Time : Three Hours

Maximum : 100 Marks

Part A

Each question carries 5 marks.

I. (a) Evaluate $\left(e^{ax} - e^{bx}\right)/x$ as $x \to 0$.

(b) Find the radius curvature of $x^4 + y^4 = 2$ at (1, 1).

(c) Test the convergence of $\dfrac{x}{1+x} - \dfrac{x^2}{1+x^2} + \dfrac{x^3}{1+x^3} - \ldots$ for $(0 < x < 1)$.

(d) Expand $\dfrac{1}{1+x}$ using Maclaurin's series.

(e) Find the first column of the following orthogonal matrix :—

$$A = \frac{1}{3}\begin{pmatrix} - & -2 & 2 \\ - & 1 & 2 \\ - & -2 & -1 \end{pmatrix}.$$

(f) Find the value of μ and λ such that the system of equations $x + y + z = 6, x + 2y + 3z = 10$ and $x + 2y + \mu z = \lambda$ have (i) no solution ; (ii) unique solution.

(g) If $e^{-x} = \dfrac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nx) + b_n \sin(nx)$ find a_0 and a_n.

(h) Find the half range Fourier cosine series for $f(x) = x$ in $0 < x < \pi$.

(8 × 5 = 40 marks)

Part B

II. (a) Find the envelope of $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, where $a^n + b^n = c^n$, a, b are parameters. (8 marks)

(b) Verify Euler's theorem for $x^3 - 2x^2y + 3xy^2 + y^3$. (7 marks)

Or

(c) Show that $\partial\left(\dfrac{u, v}{x, y}\right) = \partial\left(\dfrac{u, v}{p, q}\right) \times \partial\left(\dfrac{p, q}{x, y}\right)$ where $\partial\left(\dfrac{u, v}{x, y}\right)$ is Jacobian of u, v with respect to x and y.

(7 marks)

Turn over

(d) Find the shortest and longest distance from the point (1, 2, – 1) to the sphere $x^2 + y^2 + z^2 = 24$. (8 marks)

III. (a) Test the convergence of the series $\left(\frac{2}{3}\right) x + \left(\frac{3}{4}\right)^2 x^2 + \left(\frac{4}{5}\right)^3 x^3 + \ldots\ldots$ (8 marks)

(b) Test the convergence of the series $\left(\frac{1}{2} + \frac{1}{3}\right) + \left(\frac{1}{2^2} + \frac{1}{3^2}\right) + \left(\frac{1}{2^3} + \frac{1}{3^3}\right) + \ldots\ldots$ (7 marks)

Or

(c) Test the convergence of the series $\sum_{n=1}^{\infty} \sqrt{\frac{n}{n+1}}\, x^n$, where x is any positive number. (8 marks)

(d) State Raabe's test. Also show that for $\Sigma u_n = \Sigma \frac{1}{n}$ both Raabe's and ratio tests fail. (7 marks)

IV. (a) Using Cayley-Hamilton theorem, find A^{-1} if $A = \begin{pmatrix} 1 & 2 & -2 \\ 2 & 5 & -4 \\ 3 & 7 & -5 \end{pmatrix}$. (7 marks)

(b) Find the nature of the quadratic form $2x^2 + 3y^2 + 2z^2 + 2xy$. (8 marks)

Or

(c) Reduce the quadratic form $10x_1^2 + 2x_2^2 + 5x_3^2 + 6x_2x_3 - 10x_3x_1 - 4x_1x_3$ to a canonical form using orthogonal reduction.

(15 marks)

V. (a) In $-\pi < x < \pi$, express $\sinh(ax)$ in Fourier series of periodicity 2π. (8 marks)

(b) Find half range Fourier cosine series of $f(x) = x$ in $0 < x < \pi$. (7 marks)

Or

(c) Obtain the Fourier series of periodicity 2π for $f(x) = |x|$ when $-\pi < x < \pi$. (8 marks)

(d) Compute first two harmonics of the Fourier series for $f(x)$ from the following data :—

x	:	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
$f(x)$	:	2.34	3.01	3.68	4.15	3.69	2.20	0.83	0.51	0.88	1.09	1.19	1.64

(7 marks)

[4 × 15 = 60 marks]