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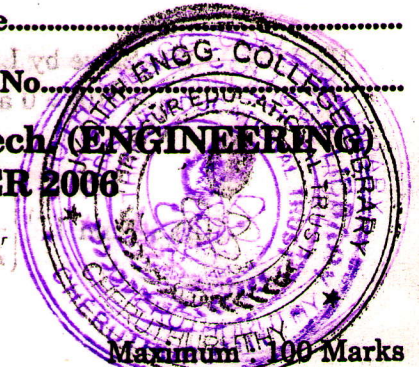
Name.....

Reg. No.....

COMBINED FIRST AND SECOND SEMESTER B.Tech. (ENGINEERING) DEGREE EXAMINATION, DECEMBER 2006

EN 2K 102—MATHEMATICS—II

(Common to all Branches)



Time : Three Hours

Maximum : 100 Marks

Answer all the questions.

- I. (a) Solve  $y dx - x dy + 3x^2y^2 e^x dx = 0$ .
- (b) Solve  $(D^2 + 4)y = \sin 2x$ .
- (c) Find the Laplace transform of  $t^2 e^{-t} \cos t$ .
- (d) Find  $L^{-1} \left[ \frac{s}{(s+1)(s+2)} \right]$ .
- (e) Prove that  $\nabla \cdot (\vec{F} \times \vec{G}) = \vec{G} \cdot (\nabla \times \vec{F}) - \vec{F} \cdot (\nabla \times \vec{G})$ .
- (f) Find the value of the constant  $a, b, c$  so that the vector :

$$\vec{F} = (x + 2y + az) \vec{i} + (bx - 3y - z) \vec{j} + (4x + cy + 2z) \vec{k}$$

is irrotational.

- (g) Apply Green's theorem in the plane and evaluate  $\oint_C [(y - \sin x) dx + \cos x dy]$  where C is the plane triangle enclosed by the lines  $y = 0, x = \frac{\pi}{2}$  and  $y = \frac{2}{\pi}x$ .
- (h) Evaluate  $\iiint_V (2x + y) dv$  where V is the region bounded by the cylinder  $z = 4 - x^2$  and the planes  $x = 0, y = 0, y = 2$  and  $z = 0$ .

(8 × 5 = 40 marks)

- II. (a) (i) Solve  $(D^2 - 4D + 4)y = e^{2x} + \cos 2x$ . (7 marks)
- (ii) Solve  $x^2 \frac{d^2y}{dx^2} - 7x \frac{dy}{dx} + 12y = x^2$ . (8 marks)

Or

- (b) (i) Find the orthogonal trajectories of the family of parabolas  $y^2 = 4ax$ . (7 marks)
- (ii) Solve  $x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + y = \log x$ . (8 marks)

Turn over

III. (a) (i) Solve by Laplace transform, the equation  $y''' + 2y'' - y' - 2y = 0$  given  $y(0) = 0$ ,  $y'(0) = 0$  and  $y''(0) = 6$ .

(7 marks)

(ii) Find  $L^{-1}\left\{\frac{p^2 - p + 2}{p(p+2)(p-3)}\right\}$ .

(8 marks)

Or

(b) (i) Find the Laplace transform of  $f(t) = \frac{kt}{p}$ , for  $0 < t < p$  and  $f(t+p) = f(t)$ . (7 marks)

(ii) Using Laplace transform, solve the initial value problem  $y'' + 2y' + 5y = e^t \sin t$ , given that  $y(0) = 0$ , and  $y'(0) = 1$ .

(8 marks)

IV. (a) (i) Find the value of "a" if  $(x + 3y)\vec{i} + (y - 2z)\vec{j} + (x + az)\vec{k}$  is solenoidal.

(7 marks)

(ii) Prove that (i)  $\text{curl grad } \phi = 0$ ; (ii)  $\text{div curl } \vec{F} = 0$ .

(8 marks)

Or

(b) (i) Find the equation of the tangent plane and normal line to the surface  $xyz = 4$  at the point  $\vec{i} + 2\vec{j} + 2\vec{k}$ .

(7 marks)

(ii) Prove that  $\nabla \times (\vec{A} \pm \vec{B}) = (\nabla \times \vec{A}) \pm (\nabla \times \vec{B})$ .

(8 marks)

V. (a) Verify Green's theorem in the XY plane for  $\int_C (xy + y^2) dx + x^2 dy$  where C is the closed curve of the region bounded by  $y = x$  and  $y = x^2$ .

(15 marks)

Or

(b) Verify Stoke's theorem for a vector field by  $\vec{F} = (x^2 - y^2)\vec{i} + 2xy\vec{j}$  in the rectangular region in the XOY plane bounded by the lines  $x = 0$ ,  $x = a$ ,  $y = 0$  and  $y = b$ .

(15 marks)

[4 × 15 = 60 marks]