D 26605 (Pages : 2) Name..............................

Reg. No..............................

COMBINED FIRST AND SECOND SEMESTER B.Tech. (ENGINEERING) DEGREE EXAMINATION, DECEMBER 2006

EN 2K 102—MATHEMATICS—II

(Common to all Branches)

Time : Three Hours Maximum : 100 Marks

Answer **all** *the questions.*

I. (a) Solve $y\,dx - x\,dy + 3x^2y^2 e^{x^3} dx = 0$.

(b) Solve $(D^2 + 4)y = \sin 2x$.

(c) Find the Laplace transform of $t^2 e^{-t} \cos t$.

(d) Find $L^{-1}\left[\dfrac{s}{(s+1)(s+2)}\right]$.

(e) Prove that $\nabla \cdot \left(\vec{F} \times \vec{G}\right) = \vec{G} \cdot \left(\nabla \times \vec{F}\right) - \vec{F} \cdot \left(\nabla \times \vec{G}\right)$.

(f) Find the value of the constant a, b, c so that the vector :

$$\vec{F} = (x + 2y + az)\,\vec{i} + (bx - 3y - z)\,\vec{j} + (4x + cy + 2z)\,\vec{k}$$

is irrotational.

(g) Apply Green's theorem in the plane and evaluate $\oint_C \left[(y - \sin x)\,dx + \cos x\,dy\right]$ where C is the plane triangle enclosed by the lines $y = 0$, $x = \pi/2$ and $y = \dfrac{2}{\pi}x$.

(h) Evaluate $\iiint_V (2x + y)\,dv$ where V is the region bounded by the cylinder $z = 4 - x^2$ and the planes $x = 0, y = 0, y = 2$ and $z = 0$.

(8 × 5 = 40 marks)

II. (a) (i) Solve $(D^2 - 4D + 4)y = e^{2x} + \cos 2x$. (7 marks)

(ii) Solve $x^2 \dfrac{d^2y}{dx} - 7x \dfrac{dy}{dx} + 12y = x^2$. (8 marks)

Or

(b) (i) Find the orthogonal trajectories of the family of parabolas $y^2 = 4ax$. (7 marks)

(ii) Solve $x^2 \dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} + y = \log x$. (8 marks)

Turn over

III. (a) (i) Solve by Laplace transform, the equation $y''' + 2y'' - y' - 2y = 0$ given $y(0) = 0$ $y'(0) = 0$ and $y''(0) = 6$.

(7 marks)

(ii) Find $L^{-1}\left\{\dfrac{p^2 - p + 2}{p(p+2)(p-3)}\right\}$. (8 marks)

Or

(b) (i) Find the Laplace transform of $f(t) = \dfrac{kt}{p}$, for $0 < t < p$ and $f(t+p) = f(t)$. (7 marks)

(ii) Using Laplace transform, solve the initial value problem $y'' + 2y' + 5y = e^t \sin t$, given that $y(0) = 0$, and $y'(0) = 1$.

(8 marks)

IV. (a) (i) Find the value of "a" if $(x + 3y)\vec{i} + (y - 2z)\vec{j} + (x + az)\vec{k}$ is solenoidal.

(7 marks)

(ii) Prove that (i) curl grad $\phi = 0$; (ii) div curl $\vec{F} = 0$. (8 marks)

Or

(b) (i) Find the equation of the tangent plane and normal line to the surface $xyz = 4$ at the point $\vec{i} + 2\vec{j} + 2\vec{k}$.

(7 marks)

(ii) Prove that $\nabla \times \left(\vec{A} \pm \vec{B}\right) = \left(\nabla \times \vec{A}\right) \pm \left(\nabla \times \vec{B}\right)$. (8 marks)

V. (a) Verify Green's theorem in the XY plane for $\oint_C \left(xy + y^2\right) dx + x^2 dy$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$.

(15 marks)

Or

(b) Verify Stoke's theorem for a vector field by $\vec{F} = \left(x^2 - y^2\right)\vec{i} + 2xy\,\vec{j}$ in the rectangular region in the XOY plane bounded by the lines $x = 0$, $x = a$, $y = 0$ and $y = b$.

(15 marks)

[4 × 15 = 60 marks]