D 26604 (Pages : 2) Name..............................

Reg. No..............................

COMBINED FIRST AND SECOND SEMESTER B.Tech. (ENGINEERING) DEGREE EXAMINATION, DECEMBER 2006

EN 2K 101—MATHEMATICS—I

(New Scheme)

[Common to all Branches]

Time : Three Hours Maximum : 100 Marks

Answer **all** *questions.*

1. (a) Evaluate $\lim_{a \to \pi/2} (\tan x)^{\cos x}$.

   (b) Find the Taylor's series expansion of $x^2y^2 + 2x^2y + 3xy^2$ in powers of $(x + 2)$ and $(y - 1)$ upto the 3rd powers.

   (c) If $u = x^2 - y^2$ and $v = xy$, find the values of $\frac{\partial x}{\partial u}, \frac{\partial x}{\partial v}, \frac{\partial y}{\partial u}$ and $\frac{\partial y}{\partial v}, \frac{\partial^2 x}{\partial u^2}$.

   (d) Discuss the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n^2 + 1}$.

   (e) Find the rank of matrix $\begin{bmatrix} 1 & 2 & -1 & -5 & 4 \\ 1 & 3 & -2 & -7 & 5 \\ 2 & -1 & 3 & 0 & 3 \end{bmatrix}$.

   (f) By Cayley Hamiltonian theorem find the A^{-1}, $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

   (g) Find the value a_0, a_n and b_n for $f(x) = 2x - x^2$, $0 < x < 3$ using Fourier series (periodicity 3).

   (h) Obtain the half-range cosine series of $f(x) = x$, $0 < x < 2$.

(8 × 5 = 40 marks)

2. (a) (i) Find the envelope of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, treating it as the envelope of its normals.

(8 marks)

   (ii) If $x = \sin^{-1}\left(\frac{x^3 - y^3}{x + y}\right)$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2 \tan u$. (7 marks)

Or

   (b) (i) Find the maximum and minimum value of $f(x, y) = \sin x \sin y \sin(x + y)$; $0 < x, y < \pi$.

(8 marks)

   (ii) Find the equation of the evolute of the curve $x = a(\cos t + t \sin t)$; $y = a(\sin t - t \cos t)$.

(7 marks)

Turn over

3. (a) (i) Determine the nature of the following series (for $x > 0$) $\sum_{1}^{\infty} \frac{x^{2n-2}}{(n+1)\sqrt{n}}$. (8 marks)

(ii) Test the convergence of $\sum_{n=1}^{\infty} \frac{n^3 + a}{2n + a}$. (7 marks)

Or

(b) (i) Test the convergence of the series $\frac{1}{3}x + \frac{1.2}{3.5}x^2 + \frac{1.2.3}{3.5.7}x^3 + \ldots$ (8 marks)

(ii) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{n!}{n^n}$. (7 marks)

4. (a) (i) Find the inverse of $\begin{pmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 0 \end{pmatrix} X = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$. (8 marks)

(ii) Verify that the matrix $A = \begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ satisfies its characteristic equation and hence find A^4.

(7 marks)

Or

(b) (i) Solve by Gauss-Elimination method :

$3x + 4y + 5z = 18$

$2x - y + 8z = 13$

$5x - 2y + 7z = 20$

(8 marks)

(ii) Diagonalize $\begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$. (7 marks)

5. (a) (i) The table of values of the function $y = f(x)$ is given below :

x	:	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
y	:	1.0	1.4	1.9	1.7	1.5	1.2	1

Find a Fourier series upto the 3rd harmonic to represent $f(x)$ in terms of x.

(8 marks)

(ii) Find the half-range cosine series for the function $f(x) = (x-1)^2$ in the interval $0 < x < 1$.

(7 marks)

Or

(b) (i) Find Fourier series of periodicity 2 for $f(x) = \begin{cases} x & \text{in } -1 < x \le 0 \\ x+2 & \text{in } 0 < x \le 1 \end{cases}$ and hence deduce the sum of $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots$ to ∞.

(8 marks)

(ii) Expand $f(x) = x \sin x$ as a cosine series in $0 < x < \pi$. (7 marks)

[4 × 15 = 60 marks]