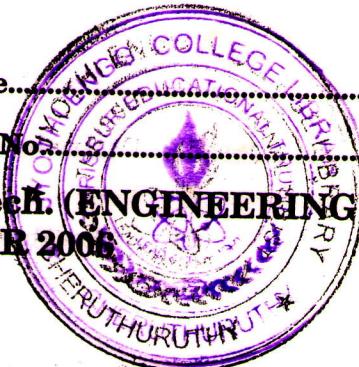


D 26604

(Pages : 2)

Name.....

Reg. No.....



**COMBINED FIRST AND SECOND SEMESTER B.Tech. (ENGINEERING)
DEGREE EXAMINATION, DECEMBER 2006**

EN 2K 101—MATHEMATICS—I

(New Scheme)

[Common to all Branches]

Time : Three Hours

Maximum : 100 Marks

Answer all questions.

1. (a) Evaluate $\lim_{x \rightarrow \frac{\pi}{2}} (\tan x)^{\cos x}$.
- (b) Find the Taylor's series expansion of $x^2y^2 + 2x^2y + 3xy^2$ in powers of $(x+2)$ and $(y-1)$ upto the 3rd powers.
- (c) If $u = x^2 - y^2$ and $v = xy$, find the values of $\frac{\partial x}{\partial u}$, $\frac{\partial x}{\partial v}$, $\frac{\partial y}{\partial u}$ and $\frac{\partial y}{\partial v}$, $\frac{\partial^2 x}{\partial u^2}$.
- (d) Discuss the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n^2 + 1}$.
- (e) Find the rank of matrix $\begin{bmatrix} 1 & 2 & -1 & -5 & 4 \\ 1 & 3 & -2 & -7 & 5 \\ 2 & -1 & 3 & 0 & 3 \end{bmatrix}$.
- (f) By Cayley Hamiltonian theorem find the A^{-1} , $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.
- (g) Find the value a_0 , a_n and b_n for $f(x) = 2x - x^2$, $0 < x < 3$ using Fourier series (periodicity 3).
- (h) Obtain the half-range cosine series of $f(x) = x$, $0 < x < 2$.

(8 x 5 = 40 marks)

2. (a) (i) Find the envelope of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, treating it as the envelope of its normals.

(8 marks)

- (ii) If $x = \sin^{-1} \left(\frac{x^3 - y^3}{x + y} \right)$ show that $x \frac{du}{dx} + y \frac{du}{dy} = 2 \tan u$.

(7 marks)

- Or
- (b) (i) Find the maximum and minimum value of $f(x, y) = \sin x \sin y \sin(x+y)$; $0 < x, y < \pi$.
 - (ii) Find the equation of the evolute of the curve $x = a(\cos t + t \sin t)$; $y = a(\sin t - t \cos t)$.

(8 marks)

(7 marks)

Turn over

3. (a) (i) Determine the nature of the following series (for $x > 0$) $\sum_{n=1}^{\infty} \frac{x^{2n-2}}{(n+1)\sqrt{n}}$. (8 marks)

(ii) Test the convergence of $\sum_{n=1}^{\infty} \frac{n^3+a}{2n+a}$. (7 marks)

Or

(b) (i) Test the convergence of the series $\frac{1}{3}x + \frac{1 \cdot 2}{3 \cdot 5}x^2 + \frac{1 \cdot 2 \cdot 3}{3 \cdot 5 \cdot 7}x^3 + \dots$ (8 marks)

(ii) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{n!}{n^n}$. (7 marks)

4. (a) (i) Find the inverse of $\begin{pmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 0 \end{pmatrix} X = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$. (8 marks)

(ii) Verify that the matrix $A = \begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ satisfies its characteristic equation and hence find A^4 . (7 marks)

Or

(b) (i) Solve by Gauss-Elimination method :

$$3x + 4y + 5z = 18$$

$$2x - y + 8z = 13$$

$$5x - 2y + 7z = 20$$

(ii) Diagonalize $\begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$. (7 marks)

5. (a) (i) The table of values of the function $y = f(x)$ is given below :

x :	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
y :	1.0	1.4	1.9	1.7	1.5	1.2	1

Find a Fourier series upto the 3rd harmonic to represent $f(x)$ in terms of x .

(8 marks)

(ii) Find the half-range cosine series for the function $f(x) = (x-1)^2$ in the interval $0 < x < 1$. (7 marks)

Or

(b) (i) Find Fourier series of periodicity 2 for $f(x) = \begin{cases} x & \text{in } -1 < x \leq 0 \\ x+2 & \text{in } 0 < x \leq 1 \end{cases}$ and hence deduce the sum of $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$ to ∞ . (8 marks)

(ii) Expand $f(x) = x \sin x$ as a cosine series in $0 < x < \pi$. (7 marks)

[4 × 15 = 60 marks]