**D 26574** 

(Pages : 2)

Name.....

Reg. No.

COMBINED FIRST AND SECOND SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, DECEMBER 2006

EN 04-101-ENGINEERING MATHEMATICS

(2004 admissions)

Time : Three Hours

Answer all questions.

Part A

- I. (a) Find the radius of curvature at the point (x, y) of the curve  $x^{2/3} + y^{2/3} = a^{2/3}$ 
  - (b) Verify Euler's theorem on homogeneous functions for the function  $u = (x^2 + y^2 + z^2)^{-\frac{1}{3}}$ .
  - (c) Discuss the convergence of the series  $\sum_{n=1}^{\infty} \left( \frac{1}{\sqrt{n}} \frac{1}{\sqrt{n+1}} \right)$ .
  - (d) Expand  $\log (1 + x)$  using Maclaurin's series.
  - (e) Find the rank of the matrix by reducing it to the normal form  $\begin{vmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \\ 9 & 10 & 11 & 19 \end{vmatrix}$
  - (f) Find the values of a and b for which the equations x + y + z = 3, x + 2y + 2z = 6, x + ay + 3z = b have (i) no solution; (ii) a unique solution.

(g) Obtain the Fourier series expansion for  $f(x) = \begin{cases} 1, & 0 < x < \frac{l}{2} \\ 0, & \frac{l}{2} < x < l \end{cases}$ (h) Expand  $f(x) = e^{-x}$  as a Fourier series in (-l, l).

 $(8 \times 5 = 40 \text{ marks})$ 

00 Marks

## Part B

II. (a) (i) Find the evolute of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ . (8 marks)

(ii) Find the centre of curvature of the cycloid  $x = a (\theta - \sin \theta), y = a (1 - \cos \theta).$  (7 marks) Or

(b) (i) The area of a triangle is calculated from the angles A and C and the side b. If  $\delta A$  is the error in measuring A, show that the relative error in the area is approximately sin C  $\delta A$ 

$$\overline{\sin A \cdot \sin (A + C)}$$
 (8 marks)

(ii) Investigate the maximum and minimum values of the expression :  $x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x.$ 

(7 marks)

Turn over

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III. (a)	(i)	Discuss the converg	ence of the serie	es $\sum_{n=1}^{\infty} \left( \frac{2^n}{2^n} \right)$	$\frac{-2}{+1}\bigg)x^{n-1}(x)$	> 0).	(8 marks
	<b>(ii)</b>	Determine the inter	val of converge	nce for the s	eries $x - \frac{x^2}{2}$	$+\frac{x^3}{3}$ -	$\frac{x^4}{4}$ +
				н. — <sup>6</sup> — — — — — — — — — — — — — — — — — — —	-		(7 marks)
			Or		1. T		
(b)	(i)	Test the convergence					(8 m <mark>arks</mark> )
`	(ii)	Discuss the converge	ence of the serie	$\sum_{n=1}^{\infty} \frac{\left(n + \frac{1}{n^n}\right)}{n^n}$	$\frac{1)^n x^n}{+1}.$	( <u>-</u>	(7 m <mark>arks</mark> )
			*		3 10	5]	
IV. (a)	(i)	Using Cayley-Hamil	ton theorem fin	ad $A^{-1}$ for $A$	= -2 -3 -	-4.	(8 marks)
1						<u>, 1</u>	
(b)	(ii) Redu	Find the nature of the cells $x^2 + 7y^2 + 3z^2 - 1$	Or $2xy - 8yz + 4xz$	into a canor			
(b)	Redu		Or $2xy - 8yz + 4xz$	into a canor			nal reduction.
	Redu	$ce 8 x^2 + 7 y^2 + 3 z^2 - 1$	Or 2 xy - 8 yz + 4xz quadratic form	; into a canor 1.	nical form by a	an orthogo	4
	Redu Also	$x = 8x^2 + 7y^2 + 3z^2 - 1$ ind the nature of the Expand $f(x) = x - x^2$ ,	$Or$ $2 xy - 8 yz + 4xz$ quadratic form $-\pi \le x \le \pi \text{ in Fc}$	z into a canor ourier series.	nical form by a	an orthogo	nal reduction.
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	Redu Also (i)	$\frac{2}{2} = 8x^{2} + 7y^{2} + 3z^{2} - 1$ ind the nature of the Expand $f(x) = x - x^{2}$ , $\frac{1}{1^{2}} - \frac{1}{2^{2}} + \frac{1}{3^{2}}$	$Or$ $2xy - 8yz + 4xz$ quadratic form $-\pi \le x \le \pi \text{ in Fo}$ $-\frac{1}{4^2} + \dots$	to into a canor a. burier series. $= \frac{\pi^2}{12}$	nical form by a	an orthogo	nal reduction. (15 marks) (8 marks)
	Redu Also	$x = 8x^2 + 7y^2 + 3z^2 - 1$ ind the nature of the Expand $f(x) = x - x^2$ ,	$Or$ $2xy - 8yz + 4xz$ quadratic form $-\pi \le x \le \pi \text{ in Fo}$ $-\frac{1}{4^2} + \dots$ $x), 0 \le x \le \pi \text{ in } z$	to into a canor a. burier series. $= \frac{\pi^2}{12}$	nical form by a	an orthogo	nal reduction. (15 marks) (8 marks)
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2