

**COMBINED FIRST AND SECOND SEMESTER B.TECH. (ENGINEERING)  
DEGREE EXAMINATION, DECEMBER 2006**

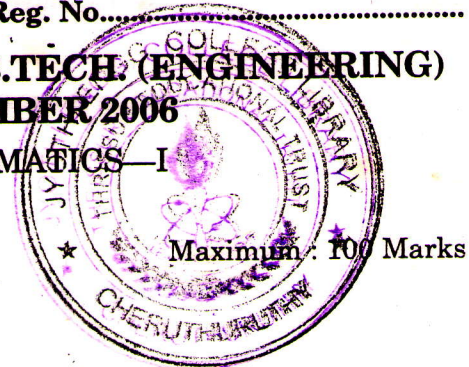
**EN 04-101—ENGINEERING MATHEMATICS—I**

(2004 admissions)

Time : Three Hours

Answer all questions.

**Part A**



- I. (a) Find the radius of curvature at the point  $(x, y)$  of the curve  $x^{2/3} + y^{2/3} = a^{2/3}$ .
- (b) Verify Euler's theorem on homogeneous functions for the function  $u = (x^2 + y^2 + z^2)^{-1/3}$ .
- (c) Discuss the convergence of the series  $\sum_{n=1}^{\infty} \left( \frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}} \right)$ .
- (d) Expand  $\log(1+x)$  using Maclaurin's series.
- (e) Find the rank of the matrix by reducing it to the normal form
- $$\begin{bmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \\ 9 & 10 & 11 & 12 \end{bmatrix}$$
- (f) Find the values of  $a$  and  $b$  for which the equations  $x + y + z = 3$ ,  $x + 2y + 2z = 6$ ,  $x + ay + 3z = b$  have (i) no solution ; (ii) a unique solution.
- (g) Obtain the Fourier series expansion for  $f(x) = \begin{cases} 1, & 0 < x < l/2 \\ 0, & l/2 < x < l \end{cases}$ .
- (h) Expand  $f(x) = e^{-x}$  as a Fourier series in  $(-l, l)$ .

(8 × 5 = 40 marks)

**Part B**

- II. (a) (i) Find the evolute of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ . (8 marks)
- (ii) Find the centre of curvature of the cycloid  $x = a(\theta - \sin \theta)$ ,  $y = a(1 - \cos \theta)$ . (7 marks)
- Or
- (b) (i) The area of a triangle is calculated from the angles  $A$  and  $C$  and the side  $b$ . If  $\delta A$  is the error in measuring  $A$ , show that the relative error in the area is approximately
- $$\frac{\sin C \delta A}{\sin A \cdot \sin(A + C)}$$
- (8 marks)
- (ii) Investigate the maximum and minimum values of the expression :
- $$x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x.$$
- (7 marks)

**Turn over**

III. (a) (i) Discuss the convergence of the series  $\sum_{n=1}^{\infty} \left( \frac{2^n - 2}{2^n + 1} \right) x^{n-1}$  ( $x > 0$ ). (8 marks)

(ii) Determine the interval of convergence for the series  $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$ . (7 marks)

Or

(b) (i) Test the convergence of the series  $\sum_{n=1}^{\infty} \frac{x^{2n-2}}{(n+1)\sqrt{n}}$  for  $x > 0$ . (8 marks)

(ii) Discuss the convergence of the series  $\sum_{n=1}^{\infty} \frac{(n+1)^n x^n}{n^{n+1}}$ . (7 marks)

IV. (a) (i) Using Cayley-Hamilton theorem find  $A^{-1}$  for  $A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$ . (8 marks)

(ii) Find the nature of the quadratic form  $3x^2 - 2y^2 - 2z^2 - 4xy + 8xz + 12yz$ . (7 marks)

Or

(b) Reduce  $8x^2 + 7y^2 + 3z^2 - 12xy - 8yz + 4xz$  into a canonical form by an orthogonal reduction. Also find the nature of the quadratic form. (15 marks)

V. (a) (i) Expand  $f(x) = x - x^2$ ,  $-\pi \leq x \leq \pi$  in Fourier series. Hence show that :

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots = \frac{\pi^2}{12}$$

(8 marks)

(ii) Expand  $f(x) = x(\pi - x)$ ,  $0 \leq x \leq \pi$  in a Fourier cosine series. (7 marks)

Or

(b) (i) Obtain the constant term and the coefficients for the first sine and cosine terms in the Fourier series that represents  $y$  as given in the following table :—

$x$	:	0	1.0	2.0	3.0	4.0	5.0	6.0
$y$	:	9	18	24	28	26	20	9

(8 marks)

(ii) Express  $f(x) = x \sin x$ ,  $0 < x < \pi$  in a half range sine series. (7 marks)

[4 × 15 = 60 marks]