C 20546

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Name...............................

Reg. No............................

SIXTH SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, JUNE 2006

CS 2K 603. GRAPH THEORY AND COMBINATORICS

Time : Three Hours | **Maximum : 100 Marks**

Answer **all** *questions.*

Part A

1. (a) Discuss the Chinese postman problem.

   (b) Define Hamiltonian graph and show that if a graph is Hamiltonian then $w\,(G - S) \leq |S|$, where $w\,(X)$ denotes the number of components of X and $|S|$ denotes the cardinality of S.

   (c) Write Kruksal's algorithm.

   (d) Show that every vertex other than degree one vertex of any tree is a cut vertex.

   (e) Enumerate the number of ways of placing 20 indistinguishable balls into five boxes, where each box is non-empty and also find the number of integral solutions to $x_1 + x_2 + x_3 + x_4 + x_5 = 20$ for $x_1 \geq 3, x_2 \geq 2, x_3 \geq 4, x_4 \geq 6$ and $x_5 \geq 0$.

   (f) If a chain letter is sent to 10 people in the first week of the year. The next week each person, who received a letter sends letters to 10 new people and so on, then find (i) how many people have received letters after 10 weeks ; (ii) at the end of the year ?

   (g) Find the generating function for a_r = the number of ways of distributing r similar balls into n numbered boxes, where each box is non-empty.

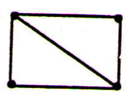
   (h) Find the generating function of $\sum_{n=0}^{\infty} n^3 a^n X^n$.

(8 × 5 = 40 marks)

Part B

2. (a) (i) Show that for every K-vertex colour critical minimum vertex degree is K – 1 at least.

(8 marks)

   (ii) Find the chromatic polynomial of 

(7 marks)

Or

   (b) (i) Show that if G is Eulerian then (1) each vertex is of even degree ; and (2) edge set of G is partitioned into disjoint cycles.

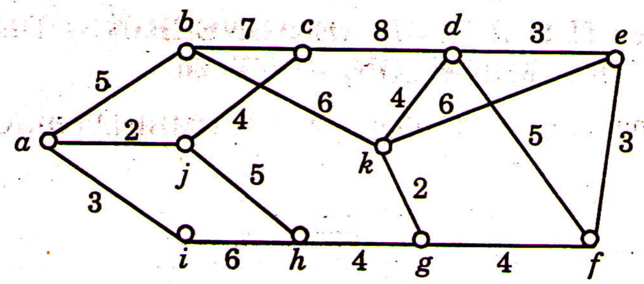
(10 marks)

   (ii) State Kurtowski's theorem and write one application.

(5 marks)

Turn over

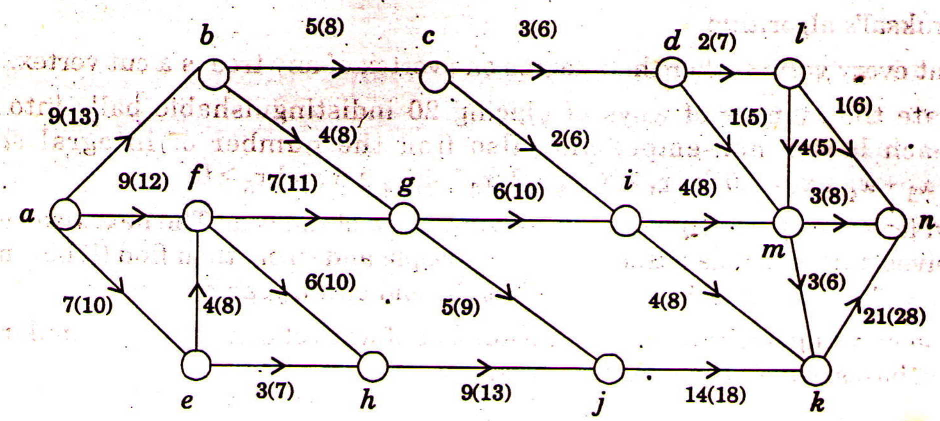
3. (a) Write Dijkstra's algorithm and implement the same from the vertex a of the following graph :—



(15 marks)

Or

(b) Write max-flow algorithm and implement the same to the following graph :—



(15 marks)

4. (a) (i) Count the number of integral solutions to $x_1 + x_2 + x_3 = 20$, where $2 \leq x_1 \leq 5$, $4 \leq x_2 \leq 7$ and $-2 \leq x_3 \leq 9$.

(8 marks)

(ii) In how many ways can 10 people arrange themselves :

(1) In a row of 10 chairs ?

(2) In a row of 7 chairs ?

(3) In a circle of 10 chairs ?

(7 marks)

Or

(b) (i) State and prove Binomial theorem. (8 marks)

(ii) If $|F| = 200$, $|R| = 50$, $|S| = 100$, $|F \cap R| = 20$, $|F \cap S| = 60$, $|R \cap S| = 35$ and $|F \cap R \cap S| = 10$, then find $|F \cup R \cup S|$.

(7 marks)

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5. (a) (i) Solve the recurrence relation using generating function $a_n - a_{n-1} = 2(n-1)$ for $n \geq 1$, and $a_0 = 2$. (8 marks)

(ii) Solve $a_n^2 - 2a_{n-1}^2 = 1$ for $n \geq 1$, where $a_0 = 2$. (7 marks)

Or

(b) (i) Solve $a_n - 6a_{n-1} + 8a_{n-2} = n4^n$, where $a_0 = 8$ and $a_1 = 22$. (7 marks)

(ii) Solve $a_n - a_{n-1} = n$ for $n \geq 1$ and $a_0 = 0$ using generating function. (8 marks)

[4 × 15 = 60 marks]