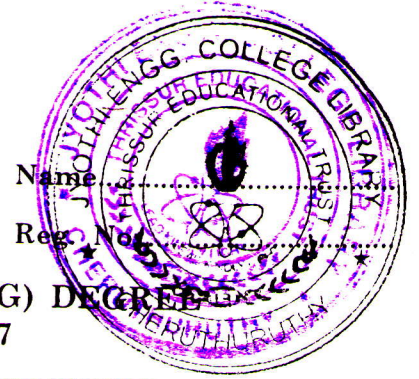


D 42029

(Pages : 2)



Name

Reg. No.

THIRD SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, DECEMBER 2007

CS/IT 04 303—DISCRETE COMPUTATIONAL STRUCTURES

(2004 Admissions)

Time : Three Hours

Maximum : 100 Marks

Answer all questions.

1. (a) What do you mean by Universal quantifiers and Existential quantifiers ?
- (b) State and prove De Morgan's laws using truth tables.
- (c) Let $A = \{a, b, c\}$. Determine whether the relation R whose matrix M_R is given is an equivalence relation.

$$M_R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

- (d) Let $A = \{1, 2, 3, 4, 12\}$. Consider the partial order of divisibility on A . That is, if a and $b \in A$, $a \leq b$ if and only if $a|b$. Draw the Hasse diagram of the poset (A, \leq) .
- (e) Write short notes on Hamming code.
- (f) What do you mean by Euler path and Euler circuit ? Illustrate with an example.
- (g) If $F(a)$ is the finite extension of F , show that a is algebraic over F .
- (h) Define Rings. Give an example.

(8 × 5 = 40 marks)

2. (a) (i) Show that $(P \rightarrow Q) \wedge (R \rightarrow Q)$ and $(P \vee R) \rightarrow Q$ are equivalent. (8 marks)
- (ii) Show that $R \wedge (P \vee Q)$ is a valid conclusion from the premises $P \vee Q, Q \rightarrow R, P \rightarrow M$ and $\neg M$.

(7 marks)

Or

- (b) (i) Prove by using direct method :

The sum of an even integer and an odd integer is an odd integer.

(7 marks)

- (ii) Prove that $\sqrt{5}$ is not a rational number. (Prove by contradiction). (8 marks)

3. (a) (i) Prove that if R is a symmetric relation, then $R \cap R^{-1} = R$. (7 marks)

- (ii) On the set of Natural number N , the relation R is defined " aRb " if and only if " a divided b ". Show the R is antisymmetric.

(8 marks)

Or

Turn over

- (b) (i) \mathbb{R} is the set of real numbers given that

$$f(x) = x + 2, g(x) = x - 2, \text{ and } h(x) = 3x \quad \forall x \in \mathbb{R}.$$

Find $G \circ F, f \circ g, f \circ f, g \circ g, f \circ h, h \circ g, h \circ f$ and $f \circ g \circ h$.

(8 marks)

- (ii) Let \leq be a partial ordering of a set s . Define the dual order on s . How is the dual order related to the inverse of the relation \leq ?

(7 marks)

4. (a) (i) Show that the set $G = \{-1, 1\}$, is a finite abelian group of order 2, under multiplication.

(7 marks)

- (ii) If $(G, *)$ is a group of even order prove that it has an element $a \neq e$ satisfying $a^2 = e$.

(8 marks)

Or

- (b) (i) State and prove Lagrange's theorem.

(10 marks)

- (ii) Show that every cyclic group is commutative.

(5 marks)

5. (a) (i) Show that the system $(\mathbb{E}, +, \cdot)$ of even integer is a ring under ordinary addition and multiplication.

(7 marks)

- (ii) If \mathbb{R} is a ring commutative with characteristic 2, show that $(a+b)^2 = a^2 + b^2 \quad \forall a, b \in \mathbb{R}$.

(8 marks)

Or

- (b) Show that an element $a \in K$ is algebraic over F if and only if $F(a)$ is finite extension of the field F .

(15 marks)

[4 × 15 = 60 marks]