D 42029 (Pages : 2) Name..................................

Reg. No...............................

THIRD SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, DECEMBER 2007

CS/IT 04 303—DISCRETE COMPUTATIONAL STRUCTURES

(2004 Admissions)

Time : Three Hours Maximum : 100 Marks

Answer **all** *questions.*

1. (a) What do you mean by Universal quantifiers and Existential quantifiers ?

(b) State and prove De Morgan's laws using truth tables.

(c) Let A = {*a*, *b*,*c*}. Determine whether the relation R whose matrix M_R is given is an equivalence relation.

$$M_R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}.$$

(d) Let A = {1, 2, 3, 4, 12}. Consider the partial order of divisibility on A. That is, if a and $b \in$ A, $a \leq b$ if and only if $a \mid b$. Draw the Hasse diagram of the poset (A, ≤).

(e) Write short notes on Hamming code.

(f) What do you mean by Euler path and Euler circuit ? Illustrate with an example.

(g) If F (a) is the finite extension of F, show that a is algebraic over F.

(h) Define Rings. Give an example.

(8 × 5 = 40 marks)

2. (a) (i) Show that $(P \to Q) \wedge (R \to Q)$ and $(P \vee R) \to Q$ are equivalent. (8 marks)

(ii) Show that $R \wedge (P \vee Q)$ is a valid conclusion from the premises $P \vee Q, Q \to R, P \to M$ and $\rceil M$.

(7 marks)

Or

(b) (i) Prove by using direct method :

The sum of an even integer and an odd integer is an odd integer.

(7 marks)

(ii) Prove that $\sqrt{5}$ is not a rational number. (Prove by contradiction). (8 marks)

3. (a) (i) Prove that if R is a symmetric relation, then $R \cap R^{-1} = R$. (7 marks)

(ii) On the set of Natural number N, the relation R is defined "aRb" if and only if "a divided b". Show the R is antisymmetric.

(8 marks)

Or

Turn over

(b) (i) R is the set of real numbers given that

$$f(x) = x + 2,\ g(x) = x - 2, \text{ and } h(x) = 3x\ \forall\ x \in \text{R}.$$

Find $G \circ F, f \circ g, f \circ f, g \circ g, f \circ h, h \circ g, h \circ f$ and $f \circ g \circ h$.

(8 marks)

(ii) Let $\leq$ be a partial ordering of a set s. Define the dual order on s. How is the dual order related to the inverse of the relation $\leq$?

(7 marks)

4. (a) (i) Show that the set G = {– 1, 1}, is a finite abelian group of order 2, under multiplication.

(7 marks)

(ii) If (G, *) is a group of even order prove that it has an element $a \neq e$ satisfying $a^2 = e$.

(8 marks)

Or

(b) (i) State and prove Lagrange's theorem. (10 marks)

(ii) Show that every cyclic group is commutative. (5 marks)

5. (a) (i) Show that the system (E, +, .) of even integer is a ring under ordinary addition a multiplication.

(7 marks)

(ii) If R is a ring commutative with characteristic 2, show that $(a+b)^2 = a^2 + b^2\ \forall\ a, b \in \text{R}$.

(8 marks)

Or

(b) Show that an element $a \in$ K is algebraic over F if and only if F (a) is finite extension of the field F.

(15 marks)

[4 × 15 = 60 marks]