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Name..............................

Reg. No..............................

THIRD SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, DECEMBER 2007

EN 04 301 A—ENGINEERING MATHEMATICS—III

(Common to all except CS/IT)

[2004 admissions] EC

Time : Three Hours Maximum : 100 Marks

Answer **all** *questions.*

1. (a) Define the followings : vector space, subspace, basis and linear dependence.

   (b) Verify all the axioms of the vector space on the set $V = \{(x, y) \in R^{(2)} : x + 2y = 10\}$.

   (c) Find the Fourier transform of $e^{-|a|t}$.

   (d) Find the Fourier sine transform of $f(x) = \begin{cases} \sin x & 0 \le x \le \pi \\ 0 & x > \pi \end{cases}$.

   (e) Derive the mean of standard Poisson distribution.

   (f) If the mean and variance of a binomial distribution are 30 and 25 respectively, find $P(X = 3)$.

   (g) Explain Null Hypothesis and alternative Hypothesis.

   (h) The weights of packing boxes are normally distributed with a standard deviation 6 gms. A sample of 100 boxes are chosen and the mean weight in 158 gms. Find the 99% confidence limits of the population mean.

   ($8 \times 5 = 40$ marks)

2. (a) (i) Show that the vectors (4, 3, – 1) (2, – 1, 5) and (–1, 1, 2) are linearly independent. Express (5, 15, 2) as a linear combination of above vectors.

   (7 marks)

   (ii) Verify whether $<u, v> = x_1 y_1 - x_2 y_1 - x_1 y_2 + x_2 y_2$ is an inner product in $R^{(2)}$ or not for $u = (x_1, x_2)$ and $v = (y_1, y_2)$.

   (8 marks)

   Or

   (b) (i) Find a basis and dimension of the subspace u in $R^{(4)}$ generated by (1, – 1, 2, 4), (2, 1, 3, 0), (1, 2, – 1, 0).

   (7 marks)

   (ii) Find the set of orthonormal vectors in $R^{(3)}$ for the basis (1, 2, 2,), (–2, 1, 0) and (3, 0, 4).

   (8 marks)

Turn over

3. (a) (i) Find the Fourier transform of $f(x) = \begin{cases} \sin ax & -\pi/a < x < \pi/a \\ 0 & \text{otherwise} \end{cases}$ (7 marks)

(ii) Find the Fourier transform of $e^{-x^2/2}$. (8 marks)

Or

(b) (i) Find the Fourier sine transform of $2e^{-5x} + 5e^{-2x}$. (7 marks)

(ii) Using Fourier integral, prove that $\int_0^\infty \frac{\cos x\lambda}{1+\lambda^2} d\lambda = \frac{\lambda e^{-x}}{2}, x \geq 0$. (8 marks)

4. (a) (i) The density function of a random variable X is given by $f(x) = \begin{cases} kx & 0 \leq x \leq 5 \\ \frac{k}{2}(15 - x) & 5 \leq x \leq 15 \end{cases}$

Find the value of k, E (X) and P $[1 \leq X \leq 10]$.

(7 marks)

(ii) The probability that a pen manufactured by a company will be defective is 0.15. A random sample of 10 pens a chosen. What is the probability that in the sample (1) not more than 1 is defective ; (2) at least 7 are good ; and (3) all are good.

(8 marks)

Or

(b) (i) Fit a Poisson distribution to the following data and calculate theoretical frequency :—

x	...	0	1	2	3	4
f	...	122	60	15	2	1

(7 marks)

(ii) In a normal distribution 7% of the items are under 35 and 10% of the items are above 55. Find mean and variance.

(8 marks)

5. (a) (i) The mean weight obtained from a random sample of size 100 is 64 gms. The standard deviation of the population is 3 gms. Test the hypothesis that the mean weight of t[he] population is 67 gms at 5% level of significance.

(7 marks)

(ii) Intelligent test was given to two groups of students :

		Mean	S.D.	Size
Group I	...	75	8	60
Group II	...	73	10	100

Test if the difference between the mean scores is significant.

(8 marks)

Or

(b) (i) The two random samples reveal the following data :

Sample No.		*Size*	*mean*	*s.d.*
I	...	1600	65	3.5
II	...	1000	63	2.7

Is the difference between the standard deviations significant ?

(7 marks)

(ii) The following table slows the distribution of digits in number chosen at random from a telephone directory :

Digit	...	0	1	2	3	4	5	6	7	8	9
Frequency	...	1026	1107	997	966	1075	933	1107	972	964	853

Using chi-square test, examine whether the digits may be taken to occur equally frequently in the directory.

(8 marks)

[4 × 15 = 60 marks]