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Name.........................

Reg. No.........................

THIRD SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, DECEMBER 2007

Computer Science Engineering/Information Technology

PTCS/IT/CS 2K 303—DISCRETE COMPUTATIONAL STRUCTURES

Time : Three Hours

Maximum : 100 Marks

Answer **all** *the questions.*

1. (a) Find the truth table of $(\neg (P \vee Q) \vee \neg R) \vee (((\neg P \vee Q) \vee \neg R) \wedge P)$.

   (b) Show that $P \rightarrow (Q \rightarrow R) \Leftrightarrow (P \wedge Q) \rightarrow R$ without using truth table.

   (c) Show that for any *two* sets A and B $A - (A \cap B) = A - B$.

   (d) Let A be the set of factors of the integer 20 and $R = \{(x, y)/x \text{ divides } y\}$. Draw the graph of R and also give its matrix.

   (e) Define $g : (z, +) \rightarrow (z_n + n)$ by $g(x)$ = remainder of x on division by n prove that g is a homomorphism.

   (f) If (G, *) is an abelian group, then for all $a, b, \in G$, show that $(a * b)^n = a^n * b^n$.

   (g) Define a Boolean ring. Prove that every boolean ring is commutative.

   (h) If R is a ring, then show that for all $a, b \in R$, (i) $a0 = 0a = 0$ (ii) $a(-b) = (-a)b = -(ab)$.

   (8 × 5 = 40 marks)

2. (a) (i) Obtain the principal conjuctive normal form of the statement $(P \wedge Q) \vee (\neg P \wedge Q \wedge R)$. (7 marks)

   (ii) Prove that $(\exists x)(P(x) \wedge Q(x)) \Rightarrow (\exists x) P(x) \wedge (\exists x) Q(x)$. (8 marks)

   Or

   (b) (i) Show that $P \rightarrow Q, P \rightarrow R, Q \rightarrow \neg R$ and P are inconsistent. (7 marks)

   (ii) Show that $P \vee Q, Q \rightarrow R, P \rightarrow M, \neg M \Rightarrow R \wedge (P \vee Q)$. (8 marks)

3. (a) (i) If $f : x \rightarrow x$ and $g : x \rightarrow x$ are both objective, show that $(g \cdot f)^{-1} = f^{-1} \cdot g^{-1}$. (8 marks)

   (ii) If A has 'n' elements, then prove that P (A) has 2^n elements. (7 marks)

   Or

Turn over

(b) (i) Show that the De Morgan's laws given by $(a * b)' = a' \oplus b'$ and $(a \oplus b)' = a' * b'$ hold in a complemented, distributive lattice.

(8 marks)

(ii) Let R be a relation on the set of integers such that a R b iff $a \leq b$. Show that R is an equivalence relation.

(7 marks)

4. (a) (i) Prove that any subgroup of a cyclic group is itself a cyclic group. (8 marks)

(ii) Show that the intersection of two normal subgroups of G is a normal subgroup of G.

(7 marks)

Or

(b) (i) If H is a subgroup of G, and $a \in$ G let $aHa^{-1} = \left\{aha^{-1}/h \in H\right\}$. Show that aHa^{-1} is a subgroup of G.

(8 marks)

(ii) Prove that the minimum weight of the non-zero code words in a group code is equal to its minimum distance.

(7 marks)

5. (a) (i) Prove that every finite integral domain is a field. (8 marks)

(ii) If ϕ is a homomorphism of a ring R into a ring R′, then show that $\phi(0) = 0$ and $\phi(-a) = -\phi(a)$ for every $a \in$ R.

(7 marks)

Or

(b) (i) Define an Euclidean ring. Prove that every Euclidean ring possesses a unit element.

(8 marks)

(ii) Prove that $x^2 + x + 4$ is irreducible over F, the field of integers mod 11. (7 marks)

[4 × 15 = 60 marks]