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Name.....................................

Reg. No..................................

THIRD SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, DECEMBER 2007

CS/IT/PTCS 2K 301—ENGINEERING MATHEMATICS—III

Time : Three Hours

Maximum : 100 Marks

Answer **all** *questions.*

I. (a) Show that $S_1 = \{(1, 0, 1), (0, 1, 1), (1, 1, 0)\}$ and $S_2 = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$ are two different bases for R^3.

(b) Show that every superset of linearly dependent set is linearly dependent.

(c) Find the rank of $\begin{pmatrix} 1 & -1 & 1 & -1 \\ 1 & -3 & 4 & -6 \\ 4 & 3 & -2 & -3 \\ 7 & -4 & 7 & -16 \end{pmatrix}$.

(d) Find the eigen values and eigen vectors of $\begin{pmatrix} 2 & 1 & -1 \\ 1 & 1 & -2 \\ -1 & -2 & 1 \end{pmatrix}$.

(e) Show that every analytic function $w = f(z)$ is free from $\overline{Z}$.

(f) Show that $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ is harmonic and if $f(z) = u + iv$ is analytic find v.

(g) Find the value of $\int_{|z+4|=6} \frac{5z+8}{(z+8)(z+2)}\, dz$.

(h) Expand $\frac{z-5}{(z+8)(z-9)}$ in Laurent's series in $|z+8| < 17$.

(8 × 5 = 40 marks)

II. (a) (i) Show that every finite dimensional vector space has an orthonormal basis. (8 marks)

(ii) Let T be defined by $T(x,y) = (x\cos\alpha - y\sin\alpha,\ x\sin\alpha + y\cos\alpha)$ in $\mathbb{R}^2$. Show that T is a linear transformation.

(7 marks)

Or

Turn over

(b) (i) Orthogonalise the vectors $v_1 = (0, 1, 0, 1)$, $v_2 = (-2, 3, 0, 1)$, $v_3 = (1, 1, 1, 5)$ in $\mathbb{R}^4$.

(10 marks)

(ii) Find the linear transformation corresponding to the real matrix $A = \begin{pmatrix} 2 & -1 \\ -2 & 4 \\ 0 & 3 \\ 1 & 0 \end{pmatrix}$.

(5 marks)

III. (a) (i) Find the eigen values of $A = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$ and hence find A^n for n is a positive integar.

(7 mark

(ii) Verify Cayler-Hamilton theorem for $A = \begin{pmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{pmatrix}$ and hence find A^{-1}. (8 marks

Or

(b) Diagonalize the matrix $A = \begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$ by means of similarity transformation. (15 marks

IV. (a) (i) Find the bilinear transformation, which maps $i, -1, 1$ from z-plane into the points 0, 1, of w-plane.

(7 marks

(ii) Determine $u + iv$, an analytic function, if $u - v = \dfrac{\cos x + \sin x - e^{-y}}{2\cos x - e^{y} - e^{-y}}$ and $f\left(\pi/2\right) = 0$.

(8 marks

Or

(b) Discuss the transformation $w = z + \dfrac{k^2}{z}$ $r = \text{cons } t$, $\theta = \text{cons } t$, $\theta = 0$, $\theta = \pi/2$ and $\theta = \pi$.

(15 marks

V. (a) (i) Evaluate $\int_C \frac{\tan(z/2)}{(z-a)^2}$ with $-2 < a < 2$, where C is the boundary of square whose sides are $x = \pm 2$ and $y = \pm 2$.

(7 marks)

(ii) Evaluate $\int \frac{z\,dz}{(z-1)(z-2)^2}$.

(8 marks)

Or

(b) (i) Evaluate $\int_0^\alpha \frac{x^2\,dx}{x^4 + a^4}$.

(8 marks)

(ii) Evaluate $\int_0^{2\pi} \frac{\sin^2\theta}{5 - 3\cos\theta}\,d\theta$.

(7 marks)

[4 × 15 = 60 marks]