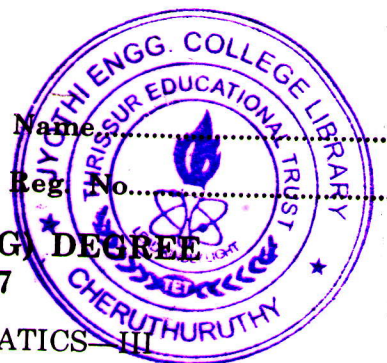


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**THIRD SEMESTER B.TECH. (ENGINEERING) DEGREE
EXAMINATION, DECEMBER 2007**

CS/IT/PTCS 2K 301—ENGINEERING MATHEMATICS—III

Time : Three Hours

Maximum : 100 Marks

Answer all questions.

1. (a) Show that $S_1 = \{(1, 0, 1), (0, 1, 1), (1, 1, 0)\}$ and $S_2 = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$ are two different bases for $\mathbb{R}^3$.
- (b) Show that every superset of linearly dependent set is linearly dependent.

(c) Find the rank of
$$\begin{pmatrix} 1 & -1 & 1 & -1 \\ 1 & -3 & 4 & -6 \\ 4 & 3 & -2 & -3 \\ 7 & -4 & 7 & -16 \end{pmatrix}$$

(d) Find the eigen values and eigen vectors of
$$\begin{pmatrix} 2 & 1 & -1 \\ 1 & 1 & -2 \\ -1 & -2 & 1 \end{pmatrix}$$

(e) Show that every analytic function $w = f(z)$ is free from $\bar{z}$.

(f) Show that $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ is harmonic and if $f(z) = u + iv$ is analytic find v .

(g) Find the value of
$$\int_{|z+4|=6} \frac{5z+8}{(z+8)(z+2)} dz.$$

(h) Expand $\frac{z-5}{(z+8)(z-9)}$ in Laurent's series in $|z+8| < 17$.

(8 × 5 = 40 marks)

- II. (a) (i) Show that every finite dimensional vector space has an orthonormal basis. (8 marks)
- (ii) Let T be defined by $T(x, y) = (x \cos \alpha - y \sin \alpha, x \sin \alpha + y \cos \alpha)$ in $\mathbb{R}^2$. Show that T is a linear transformation.

(7 marks)

Or

Turn over

(b) (i) Orthogonalise the vectors $v_1 = (0, 1, 0, 1)$, $v_2 = (-2, 3, 0, 1)$, $v_3 = (1, 1, 1, 5)$ in $\mathbb{R}^4$.

(10 marks)

(ii) Find the linear transformation corresponding to the real matrix $A = \begin{pmatrix} 2 & -1 \\ -2 & 4 \\ 0 & 3 \\ 1 & 0 \end{pmatrix}$.

(5 marks)

III. (a) (i) Find the eigen values of $A = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$ and hence find A^n for n is a positive integer.

(7 marks)

(ii) Verify Cayley-Hamilton theorem for $A = \begin{pmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{pmatrix}$ and hence find A^{-1} .

(8 marks)

Or

(b) Diagonalize the matrix $A = \begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$ by means of similarity transformation. (15 marks)

IV. (a) (i) Find the bilinear transformation, which maps $i, -1, 1$ from z -plane into the points $0, 1, \infty$ of w -plane.

(7 marks)

(ii) Determine $u + iv$, an analytic function, if $u - v = \frac{\cos x + \sin x - e^{-y}}{2 \cos x - e^y - e^{-y}}$ and $f\left(\frac{\pi}{2}\right) = 0$.

(8 marks)

Or

(b) Discuss the transformation $w = z + \frac{k^2}{z}$ $r = \text{const } t$, $\theta = \text{const } t$, $\theta = 0$, $\theta = \pi/2$ and $\theta = \pi$.

(15 marks)



V. (a) (i) Evaluate $\int_C \frac{\tan\left(\frac{z}{2}\right)}{(z-a)^2}$ with $-2 < a < 2$, where C is the boundary of square whose sides are $x = \pm 2$ and $y = \pm 2$. (7 marks)

(ii) Evaluate $\int \frac{zdz}{(z-1)(z-2)^2}$. (8 marks)

Or

(b) (i) Evaluate $\int_0^a \frac{x^2 dx}{x^4 + a^4}$. (8 marks)

(ii) Evaluate $\int_0^{2\pi} \frac{\sin^2 \theta}{5 - 3 \cos \theta} d\theta$. (7 marks)

[4 × 15 = 60 marks]