D 41947 (Pages 3) Name..............................

Reg. No..............................

THIRD SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, DECEMBER 2007

Chemical Engineering

CH 2K 301—ENGINEERING MATHEMATICS

(Common to CH/EC/CE/EE/AI/IC/PE/ME/BM/BT/PT/PTEE/PTME/PTCE/PTCH/PTEC)

Time : Three Hours Maximum : 100 Marks

Answer **all** *questions.*

I. (a) Prove that the vectors (1, 2, 3) (2, 3, – 1), (1, – 1, 1) are independent vectors in R^3.

(b) Find the subspace u in R^3 spanned by the vectors (1, – 2, 1) (–2, 0, 3), (3, – 2, – 2).

(c) Find the rank of the matrix $\begin{pmatrix} 1 & 2 & -1 & 3 & 0 \\ 2 & -1 & 1 & 2 & 1 \\ 0 & 5 & -3 & 4 & -1 \\ 4 & 3 & -1 & 8 & 1 \end{pmatrix}$.

(d) If $f(x) = x/(x+5)$, compute $f(A)$ for $A = \begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix}$.

(e) Derive the mean and variance of distribution.

(f) If 8% of the rivets produced by a machine and defective, find the probability that out of 10 rivets chosen at random (i) less than 2 will be defective ; (ii) exactly 5 will be defective.

(g) The mean repair time and standard deviation of a random sample of 50 machines are 15.5 minutes and 2.1 minutes respectively. Construct a 0.95 confidence interval for the true mean repair time.

(h) A random sample of 900 items has a mean 3.4 and standard deviation 2.61. Is the sample from a large population of mean 3.25 ? Level of significance is 5%.

($8 \times 5 = 40$ marks)

II. (a) Find the basis and the dimension of the vector space V spanned by the vectors (3, 9, 3, 5), (4, 12, 4, 5) (2, 6, 1, 0), (5, 15, 3, 2).

(7 marks)

Turn over

(b) Verify wether the following transformations are linear or not :

(i) $T : R^3 \to R^4$ where $T(x, y, z) = (x + y, y + 3z, z + x, 0)$

(ii) $T : R^3 \to R^3$, where $T(x, y, z) = (x + 2yz, 2y + 3z, 3z + x)$.

(8 marks)

Or

(c) If $T : R^3 \to R^3$ is a linear transformation given by $V_1 = (1, 1, 1)$, $V_2 = (1, 2, 0)$ $V_3 = (0, 1, 2)$, $TV_1 = (3, 3, 3)$, $TV_2 = (3, 4, 0)$, $TV_3 = (3, 4, 6)$, find $T(x, y, z)$.

(7 marks)

(d) Find the inverse of the linear transformation on R^3 for the linear transformation $T(x, y, z) = (x - y + z, x + 2y, y - z)$.

(8 marks)

III. (a) Find a pair matrices (P, Q) such that PAQ is a diagonal matrix for $A = \begin{pmatrix} 1 & -1 & 2 \\ 2 & 1 & 2 \\ 0 & 1 & 3 \end{pmatrix}$.

(7 marks)

(b) Find a transformation which will reduce the quadratic from

$$2x_1 x_2 + 6x_2 x_3 + 2x_1 x_3 + x_1^2 + 5x_2^2 + 3x_3^2$$

to a sum of squares.

(8 marks)

Or

(c) Find the characteristic values and characteristic vector of the matrix $\begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix}$.

(7 marks)

(d) If $f(x) = x^2 - 4x + 3$ and $A = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$, prove that the characteric values of $f(A)$ are equal to $f(\lambda_i)$.

(8 marks)

IV. (a) A certain screw making machine produces an average of 2 defective screws out of 100 ar pack them in boxes of 500. Find the probability that a randomly chosen box contai (i) 12 defectives ; (ii) less than 5 defectives.

(7 mark

(b) Derive the expression for E(X) and $E(X^2)$ of gamma distribution. (8 mark

Or

(c) A random variable has a normal distribution with mean $\mu = 85.0$ and standard deviatic $\sigma = 4.7$. What are the probabilities that the random variable takes a value (i) less than 87 (ii) between 81 and 90.

(7 mark

(d) If the life, in years of a electronic device has a Weibull distribution with parameters $\alpha = 0.$ and $\beta = 2$, find (i) the probability that the life of the device exceeds 5 years ; (ii) the mean lif time.

(8 marks

V. (a) In six test sums it took 14, 12, 13, 16, 12 and 11 minutes to assemble a certain mechanica device. Construct a . 95 confidence interval for σ, the true standard deviation of the amoun of time it taken to assemble the mechanical device.

(7 marks

(b) A sample of 100 bulbs of brand A gave a mean life time of 1100 hours and standard deviatio 80 hours ; another sample of 150 bulbs of brand B gave a mean life time of 1,300 hours an standard deviation 90 hours. Examine whether the difference between means is significan or not.

(8 marks)

Or

(c) Explain briefly on (i) operating characteristic curves ; (ii) Randomization. (7 marks)

(d) Fit a Poisson distribution for the following data and test the goodness of fit :

x	...	0	1	2	3	4
f	...	135	72	21	7	1

(8 marks)

[4 × 15 = 60 marks]