D 32019 (Pages : 2) Name.....

Reg. No.....

SIXTH SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION
JUNE 2007

CS 04. 604—GRAPH THEORY AND COMBINATORICS

(2004 admissions)

Time : Three Hours Maximum : 100 Marks

Answer **all** *questions.*

I. (a) Define a complete graph, regular graph and a bipartite graph.

(b) Define the term "Directed graph". When do you say two graphs are isomorphic ?

(c) Distinguish between a directed tree and a rooted tree.

(d) Define radius and diameter of a tree.

(e) Find the number of permutations of the 26 letters of the alphabet that contain the string TOPEKA.

(f) A man has 10 friends. In how many ways can he go to dinner with two or more of them.

(g) Write the generating function of the numeric function $a_r = 2^r + 3^r\ r \geq 0$.

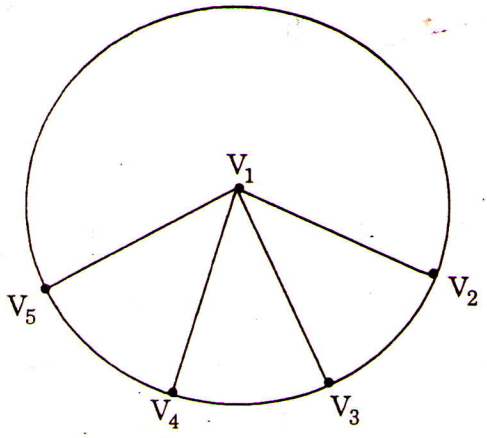
(h) Obtain the particular solution of the difference equation $ar - 5ar - 6ar - 2 = 2^r + r$.

(8 × 5 = 40 marks)

II. (a) (i) Prove that a connected graph G is Eulerian if and only if all the vertices of G are of even degree.

(8 marks)

(ii) Find the chromatic polynomial for the following graph :—



(7 marks)

Or

(b) (i) How is Travelling Salesman problem related to Hamiltonian circuits ? (8 marks)

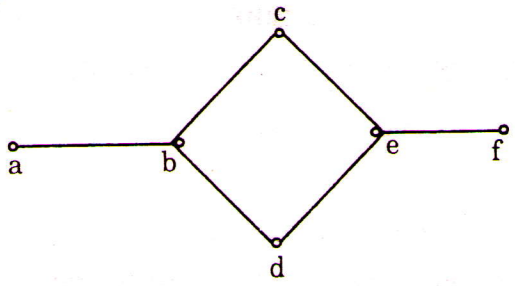
(ii) Prove that a connected graph with 'n' vertics and 'e' edges has $e - n + 2$ number of regions.

(7 marks)

Turn over

III. (a) (i) Prove that "a graph is a tree if and only if it is minimally connected. (8 marks)

(ii) Obtain the different spanning trees of the graph given below :



(7 marks)

Or

(b) (i) Show that a tree with 'n' vertices will have $(n-1)$ edges. (8 marks)

(ii) State and prove the maximum-flow and minimum cut theorem. (7 marks)

IV. (a) (i) In how many ways can a group of 8 people be divided into committees, subject to the constraint that each person must belong to exactly are committee and each committee must contain atleast two people.

(8 marks)

(ii) In how many ways can we place 'r' red balls and 'w' white balls in 'n'boxes so that each box contains atleast one ball of each colour ?

(7 marks)

Or

(b) (i) Find the number of ways in which 'm' men and 'n' women can be seated in a row so that no two women sit together.

(8 marks)

(ii) Find the number of ways in which 9 different balls can be put into 5 boxes, four of them contain 2 balls each and fifth only one.

(7 marks)

V. (a) (i) Solve the following recurrence relation $a_r - 7a_{r-1} + a_{r-2} = 3^r$ given that $a_0 = 0$ and $a_1 = 1$.

(8 marks)

(ii) Determine the discrete numeric function corresponding to the following generating function :

$$G(z) = \frac{1}{5 - 6z + z^2}.$$

(7 marks)

Or

(b) (i) Solve the recurrence relation $a_r + a_{r-1} = 3_r\ 2^r$. (8 marks)

(ii) Determine the numeric function corresponding to the generating function

$$G(z) = \frac{1 + z^2}{4 - 4z + z^2}.$$

(7 marks)