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Name.................................

Reg. No.............................

COMBINED FIRST AND SECOND SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, JUNE 2007

EN 04 102—MATHEMATICS—II

(2004 admissions)

Time : Three Hours

Maximum : 100 Marks

Answer **all** *questions.*

Part A

I. (a) Solve $\frac{dy}{dx} = \frac{x^2 + xy + y^2}{x^2}$.

(b) $(D^2 + 9)\, y = \cos 3x$.

(c) Find $L(\cosh t \sin(2t))$.

(d) Find $L^{-1}\left[\frac{e^{-\pi s}}{s^2 + 2s + 2}\right]$.

(e) Show that $\text{div}\left(\frac{\vec{r}}{r}\right) = \frac{2}{r}$.

(f) Find curl curl $\vec{f}$ at the point (1, 1, 1) if $\vec{f} = x^2 y \vec{c} + xz \vec{j} + 2yz \vec{k}$.

(g) If $\vec{F} = (3x^2 + 6y)\vec{i} - 14yz\vec{j} + 20xz^2\vec{k}$, evaluate $\int_c \vec{F} \cdot d\vec{r}$ from (0, 0, 0) to (1, 1, 1) along the curve $x = t, y = t^2, z = t^3$.

(h) Evaluate $\iint_S (x^3\, dy\, dz + x^2 y dz dx + x^2 z\, dx dz)$, where $x = 0; y = 0; z = 0; x = 1; y = 1; z = 1$.

(8 × 5 = 40 marks)

Part B

II. (a) (i) Solve $(7x - 3y - 7)\,dx + (3x - 7y - 3)\,dy = 0$. (7 marks)

(ii) Solve $2y \cos(y^2)\frac{dy}{dx} - \frac{2}{x+1}\sin(y^2) = (x+1)^3$. (8 marks)

Or

(b) (i) Solve $(D^2 - 5D + 6)\, y = e^x \cos(2x)$. (7 marks)

(ii) $x^2 y'' - xy' + 4y = \cos(\log x) + x \sin(\log x)$. (8 marks)

Turn over

III. (a) Solve $\frac{d^2y}{dx^2} + 2\frac{dy}{dt} + y = e^{-t}$ subject to the conditions $y(t) = 0$ and $\frac{dy}{dt} = 1$ at $t = 0$ using Laplace transform.

(15 marks)

Or

(b) (i) Solve $\frac{d^2y}{dt^2} + ay = \cos(2t)$ subject to the conditions $y(0) = 1$ and $y\left(\frac{\pi}{2}\right) = -1$. (7 marks)

(ii) Find $L^{-1}\left[\frac{1}{s(s+2)^3}\right]$. (8 marks)

IV. (a) (i) Determine $f(r)$, so that the vector $f(r)\vec{r}$ is both solenoidal and irrotational.

(7 marks)

(ii) Prove that $\nabla^2 r^n = (n)(n+1) r^{n-2}$, where $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ and $r = |\vec{r}|$. (8 marks)

Or

(b) (i) Prove that curl curl $\vec{F} = \text{grad}(\text{div } F) - \nabla^2 F$ and hence deduce that curl curl curl curl $\vec{F} = \nabla 4 \vec{F}$ if $\vec{F}$ is solenoidal.

(8 marks)

(ii) Prove that $\vec{F} = 3y^4z^2\vec{i} + 4x^3z^2\vec{j} - 3x^2y^2\vec{k}$ is a solenoidal vector while

$\vec{A} = 2xy\,e^z\vec{i} + x^2e^z\vec{j} + x^2y\,e^z\vec{k}$

is a irrotational vector.

(7 marks)

V. (a) Verify Stoke's theorem for $\vec{f} = (x^2 - y^2)\vec{i} + 2xy\vec{j}$ in the rectangular region $x = 0, y = 0, x = a, y = b$.

(15 marks)

Or

(b) Verify Gauss divergence theorem for $\vec{f} = (x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k}$ taken over the rectangular parallelepiped $0 \le x \le a$; $0 \le y \le b$, $0 \le z \le c$.

(15 marks)

[4 × 15 = 60 marks]