C 31697 (Pages : 2) Name..............................

Reg. No..............................

COMBINED FIRST AND SECOND SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, JUNE 2007

EN. 04 101—MATHEMATICS—I

(2004 admissions)

Time : Three Hours Maximum : 100 Marks

Answer **all** *questions.*

Part A

I. (a) If $u = \log(\tan x + \tan y + \tan z)$, then prove that $\sum \sin 2x \frac{\partial u}{\partial x} = 2$.

(b) Find the value of $\lim_{x \to \frac{\pi}{2}} (\sin x)^{\tan x}$.

(c) Test the convergence of $\frac{\sqrt{n+1} - \sqrt{n-1}}{n^3}$.

(d) Using Maclaurin's series expand $\cos x$.

(e) Does the following system of equations possess a non-zero solution ?

$$\begin{aligned} x + 2y - 3z &= 0 \\ 2x - 3y + z &= 0 \\ 4x - y - 2z &= 0 \end{aligned}$$

(f) If two of the eigenvalues of $\begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$ are 2 and 8, find the determinant.

(g) Express $\sinh ax$ in Fourier series where $-\pi < x < \pi$ with period 2π.

(h) Find a_0 and a_n if $f(x) = e^x$ in $(-\pi, \pi)$ with period 2π is $\sum_{n=0}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin(nx)$.

(8 × 5 = 40 marks)

II. (a) Find the evolute of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. (15 marks)

Or

(b) (i) Verify Euler's formula for $f(x, y) = \frac{1}{\sqrt{x^2 + y^2}}$. (7 marks)

(ii) Find the maximum of $x^n y^n z^p$ when $x + y + z = a$. (8 marks)

Turn over

III. (a) (i) Test the convergence of the series $\sum \frac{1.3.5 \ldots (2n-1)}{2.4.6.8 \ldots 2n} x^n$ using Raabe's test. (8 marks)

(ii) Show that n^{th} derivative of $x^{n-1} \log x$ is $\frac{(n-1)!}{x}$. (7 marks)

Or

(b) (i) Test the convergence of $\sum_{n=1}^{\infty} \frac{x^n}{1+x^{2n}}$. (8 marks)

(ii) Discuss the convergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} n}{2n-1}$. (7 marks)

IV. (a) Raduce the quadratic form $10x^2 + 2y^2 + 5z^2 + 6yz - 10zx - 4xy$ to canonical form by an orthogonal reduction and the rank, index, signature and nature of the quadratic form. (15 marks)

Or

(b) (i) Using Cayley-Hamilton theorem, find A^{-1} if $A = \begin{pmatrix} 1 & 3 & 4 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{pmatrix}$. (7 marks)

(ii) Find the nature of quadratic form $2x^2 + 3y^2 + 2z^2 + 2xy$. (8 marks)

V. (a) (i) Obtain cosine series for $f(x) = \begin{cases} \cos(x) & \text{in } 0 < x < \pi/2 \\ 0 & \text{in } \pi/2 < x < \pi \end{cases}$. (8 marks)

(ii) Find Fourier series of periodicity 2 for $f(x)$ given $f(x) = \begin{cases} 0 & \text{in } -1 < x < 0 \\ 1 & \text{in } 0 < x < 1 \end{cases}$. (7 marks)

Or

(b) (i) Find the first two harmonics of Fourier series of $y = f(x)$

x	...	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°
$f(x)$	...	298	356	373	337	254	155	80	51	60	93	147	221

(8 marks)

(ii) Find the Fourier series of periodicity 2π for $f(x) = x^2$ in $-\pi < x < \pi$. (7 marks)