C 31665 (Pages 3) Name.....................................

Reg. No.................................

FIRST AND SECOND SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, JUNE 2007

EN 2K 101—MATHEMATICS—I

(New Scheme)

[Common to all Branches]

Time : Three Hours Maximum : 100 Marks

Answer **all** *questions.*

I. (a) Expand $e^{\cos x}$ upto the term containing x^4 using Maclaurin's series.

(b) If $x = r\cos\theta, y = r\sin\theta$, verify that $\dfrac{\partial(x, y)}{\partial(r, \theta)} \times \dfrac{\partial(r, \theta)}{\partial(x, y)} = 1.$

(c) Find $\dfrac{dy}{dx}$, when (i) $x^3 + y^3 = 3\,ax^2$ and (ii) $x^y + y^x = C$.

(d) Discuss the convergence or divergence of the series $\sum\limits_{n=1}^{\infty} \sin\left(\dfrac{1}{n}\right)$.

(e) Find the sum of the eigen values of the inverse of $A = \begin{bmatrix} 3 & 0 & 0 \\ 8 & 4 & 0 \\ 6 & 2 & 5 \end{bmatrix}$.

(f) Show that the eigen vector of the matrix $A = \begin{bmatrix} a & b \\ -b & a \end{bmatrix}$ are $\begin{bmatrix} 1 \\ i \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -i \end{bmatrix}$.

(g) Find the Fourier series of $f(x) = \begin{cases} -k, & -\pi < x < 0 \\ k, & 0 < x < \pi, \text{ if } f(x) \end{cases}$ is periodic with period 2π.

(h) Find half range sine series for $f(x) = x$ of periodicity $2l$ in the range $0 < x < l$.

($8 \times 5 = 40$ marks)

II. (a) (i) In the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, show that the radius of curvature at the end of the major axis is equal to the semilatus rectum.

(8 marks)

(ii) If $u = f(x, y)$, where $x = r\cos\theta$ $y = r\sin\theta$, prove that $\left(\dfrac{\partial u}{\partial x}\right)^2 + \left(\dfrac{\partial u}{\partial y}\right)^2 = \left(\dfrac{\partial u}{\partial r}\right)^2 + \dfrac{1}{r^2}\left(\dfrac{\partial u}{\partial \theta}\right)^2.$

(7 marks)

Or

Turn over

(b) (i) Find the minimum value of $x^2 + y^2 + z^2$, when $x + y + z = 3a$. (8 marks)

(ii) Find the envelop of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where a and b are connected by the relation $a^2 + b^2 = c^2$, c being a constant.

(7 marks)

III. (a) (i) Determine the nature of the following series (for $x > 0$) :—

$$\sum_{n=1}^{\infty} \frac{x^{2n-2}}{(n+1)\sqrt{n}}.$$

(8 marks)

(ii) Test the convergence of $\sum_{n=1}^{\infty} \frac{n^3 + a}{2^n + a}$. (7 marks)

Or

(b) (i) Prove that the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if $p > 1$ and divergent when $p \leq 1$.

(8 marks)

(ii) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{n!}{n^n}$. (7 marks)

IV. (a) (i) Find the inverse of $\begin{pmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 0 \end{pmatrix} X = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$. (8 marks)

(ii) Verify that the matrix $A = \begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ satisfies its characteristic equation and hence find A^4.

(7 marks)

Or

(b) (i) Define :

1 Hermitian matrix ;

2 Skew-Hermitian matrix,

3 Unitary matrix ;

4 Rank of a matrix.

With examples for each.

(8 marks)

(ii) Diagonalize $\begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$. (7 marks)

V. (a) (i) Expand $f(x) = x^2, -\pi < x < \pi$. Given that $f(x)$ is periodic with period 2π and hence deduce that :

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \ldots = \frac{\pi^2}{12}.$$

(8 marks)

(ii) Find half-range Fourier cosine series for $f(x) = x$ in $0 < x < \pi$. (7 marks)

Or

(b) (i) The table of values of the function $y = f(x)$ is given below :

x	...	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
y	...	1.0	1.4	1.9	1.7	1.5	1.2	1

Find a Fourier series upto the third harmonic to represent $f(x)$ in terms of x.

(8 marks)

(ii) Expand $f(x) = x \sin x$ as a cosine series in $0 < x < \pi$. (7 marks)

[4 × 15 = 60 marks]