D 51554 **(Pages 3)** Name..................................

Reg. No..................................

FIFTH SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, DECEMBER 2008

IT 2K 503—INFORMATION THEORY AND CODING

Time : Three Hours Maximum : 100 Marks

Answer **all** *questions.*

I. (a) A discrete source emits one of 5 symbols once every millisecond with probabilities $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$ and $\frac{1}{16}$ respectively. Determine the source entropy and information rate.

(b) For the given channel matrix, compute the mutual information I (X, Y) with uniform input distribution.

$$\begin{bmatrix} 2/3 & \frac{1}{3} & 0 \\ 0 & 1/6 & 5/6 \end{bmatrix}.$$

(c) Explain error detection and correction capabilities of Hamming codes.

(d) What are the advantages and disadvantages of cyclic codes ?

(e) Define minimal polynomial, $\phi(x)$ of a field element and show that it is irreducible.

(f) Construct the vector space of all 3-tuples over a finite field with elements {0, 1} and form a 2-dimensional subspace and its dual space.

(g) Define transfer function matrix and explain in convolutional code.

(h) What is meant by catastrophic code ? Explain.

(8 × 5 = 40 marks)

II. (a) (i) Define discrete entropy, H(x), of a source with symbols $\{x_1, x_2, ..., X_M\}$. Show that H(X) is maximum when all the source symbols are equiprobable.

(11 marks)

(ii) What is meant by lossless and noiseless channel ? What is channel capacity of them ?

(4 marks)

Or

Turn over

(b) Consider a source with symbols $\{x_1, x_2, x_3, x_4\}$ and its corresponding probabilities $\{1/2, 1/4, 1/8, 1/8\}$. Construct extension of source with 2 symbols and design a code for the new symbols using (i) Shannon-Fano coding procedure and (ii) Huffman coding procedure. Compare their coding efficiencies.

(7 + 8 = 15 marks)

III. (a) The generator matrix for a (6, 3) linear block code is given by :

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

(i) Obtain all the code words and parity check matrix H. (4 marks)

(ii) What is the minimum distance of the code ? (1 mark)

(iii) Construct standard array for this code and hence determine syndromes for the coset leaders. (10 marks)

Or

(b) (i) Let $g(x)$ be the generator polynomial of an (n, k) cyclic code. Show that a binary polynomial of degree $n - 1$ or less is a code polynomial if and only if it is multiple of $g(x)$. (8 marks)

(ii) Draw the syndrome calculation circuit for a (7, 4) cyclic code with generator polynomial $g(x) = 1 + x + x^3$. Evaluate the syndrome for the received sequence 1001101 using circuit. (7 marks)

IV. (a) Construct a table for the finite field GF (2^4) based on the primitive polynomial $p(x) = 1 + x + x^4$. Find the minimal polynomial of the element α^3. Assume α be the primitive element. (15 marks)

Or

(b) (i) Let 'α' be a primitive element of the Galois field GF (2^3) such that $1 + \alpha + \alpha^3 = 0$. Find the generator polynomial and generator matrix of the single-error correcting BCH code. (10 marks)

(ii) What is meant by error location polynomial ? Explain. (5 marks)

V. (a) The transfer function of the (3, 2, 1) convolutional code is given by :

$$G(D) = \begin{bmatrix} 1+D & D & 1+D \\ D & 1 & 1 \end{bmatrix}$$

(i) Construct encoder block diagram.

(ii) Find generator matrix.

(iii) Find the code polynomial for the input polynomials $U^{(1)}(D) = 1 + D^2$ and $U^2(D) = 1 + D$.

(iv) Construct state diagram.

(3 + 3 + 3 + 6 = 15 marks)

Or

(b) (i) Explain viterbi algorithm for decoding of convolutional code. (6 marks)

(ii) Draw the trellis diagram of a rate $-\frac{1}{2}$, constraint length – 3 convolutional code with generator sequences : $g^{(1)} = (1\ 1\ 1)$ and $g^{(2)} = (1\ 0\ 1)$, viterbi algorithm, compute the decoded sequence for the received sequence.

{11, 10, 00, 01, 10, 01, 11}.

(9 marks)

[4 × 15 = 60 marks]