D 51488 (Pages 2) Name..............................

Reg. No..............................

THIRD SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, DECEMBER 2008

CS/IT 04 303—DISCRETE COMPUTATIONAL STRUCTURES

(2004 Admissions)

Time : Three Hours Maximum : 100 Marks

Answer **all** *questions.*

Part A

I. (a) Construct the truth table of the statement : $\rceil(P \vee (Q \wedge R)) \rightleftharpoons (P \vee Q) \wedge (P \vee R)$.

(b) Obtain the disjunctive normal form of $P \wedge (P \rightarrow Q)$.

(c) Define composition of relations with an example.

(d) Determine whether the operation * on the set of natural number given by $a * b = \text{v}\ \dfrac{a+b}{ab}$ is a binary operation.

(e) If the inverse of a is a^{-1}, then prove that the inverse of a^{-1} is 'a'.

(f) Define Group code.

(g) Give an example of a ring without zero divisors.

(h) State the Division Algorithm for polynomials over a field.

(8 × 5 = 40 marks)

Part B

II. (a) (i) Show that $(P \rightarrow (Q \rightarrow R)) \Rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow R))$. (7 marks)

(ii) Show that $(x)\,(P(x) \vee Q(x)) \Rightarrow (x)\,P(x) \vee (\exists x)\,Q\,(x)$. (8 marks)

Or

(b) (i) Find a conjunctive normal form of $(q \vee (p \wedge v)) \wedge \rceil((p \vee r) \wedge q)$. (7 marks)

(ii) Prove that if $(x)(P(x) \rightarrow Q(x)), (\exists y)\,P(y)$ then $(\exists z)\,Q(y)$. (8 marks)

III. (a) (i) If $f : Z \times Z \rightarrow Z$, where Z is the set of integers and $f(x, y) = x * y = x + y - xy$, show tha the binary operation * is commutative and associative.

(7 marks

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(ii) If R is the relation on the set of integers such that $(a, b) \in R$ iff $3a + 4b = 7n$ for some integer, prove that R is an equivalence relation.

(8 marks)

Or

(b) (i) If the relations R and S on a set A are represented by the matrices :

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, M_S = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

What are the matrices representing $R \cup S$ and $R \cap S$?

(7 marks)

(ii) If $F : A \to B$ and $g : B \to C$ are bijections, prove that $g \circ f : A \to C$ is also a bijection.

(8 marks)

IV. (a) (i) If R is the additive group of real numbers and R_+ is the multiplicative group of positive real numbers, prove that the mapping $f : R \to R_+$ defined by $f(x) = e^x$ for all $x \in R$ is an isomorphism.

(7 marks)

(ii) Show that (2, 5) encoding function defined by e (00) = 00000, e (01) = 01110 e (10) = 10101, e (11) = 11011 is C group code.

(8 marks)

Or

(b) (i) State and prove Lagrange's Theorem. (7 marks)

(ii) Prove that the set of all n n^{th} roots of unity forms a finite abelian group of order n wit. respect to multiplication.

(8 marks)

V. (a) (i) Show that every field is an integral domain. (7 marks)

(ii) Show that the set of all 2 × 2 non-singular matrices over rationals is not a ring under matrix addition and multiplication.

Or

(b) (i) Show that the set of numbers of the form $a + b\sqrt{2}$ with a and n as rational numbers is a field.

(7 marks)

(ii) If D is an integral domain, then prove that the polynomial ring D[x] is also an integral domain.

(8 marks)

[4 × 15 = 60 marks]