D 51436 **(Pages : 2)** Name.....................................

Reg. No.....................................

THIRD SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION DECEMBER 2008

CS/IT/PTCS 2K 303—DISCRETE COMPUTATIONAL STRUCTURES

Time : Three Hours Maximum : 100 Marks

Part A

Answer **all** *questions.*

I. (a) Prove that $(p \to q) \wedge (q \to R) \Rightarrow p \to R$.

(b) Show that $(\forall x)(p(x) \cap q(x)) \leftrightarrow (\forall(x) p(x) \cap (\forall x) q(x)$ is a logically valid statement.

(c) Prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

(d) Let $S = \{1, 2, 3, 4, 5\}$ and $A = S \times S$. The relation R on A is defined by (a,b) R (a',b') if and only if $ab' = a'b$. Prove that R is an equivalence relation.

(e) Show that $(Z_5, +_5)$ is an Abelian group.

(f) If $f : G \to G_1$ is a group homomorphism then, prove that $f(a^{-1}) = (f(a))^{-1}$.

(g) $(Z, +, \cdot)$ is not a field, give reason.

(h) Prove that the set of numbers of the form $a + b\sqrt{2}$, where a and b are integers together with ordinary addition and multiplication is a field.

(8 × 5 = 40 marks)

Part B

II. (a) (i) Show that D is a valid conclusion from the premises $(A \to B) \cap (A \to C), \neg(B \cap C), D \cup A$

(7 marks)

(ii) Prove that $\neg p\,(a, b)$ follows logically from $(\forall x)(\forall y)(p(x,y) \to w\,(x,y)$ and $\neg w(a,b)$

(8 marks)

Or

(b) (i) Show that $(S \cup R)$ is tautologically implied by $(p \cup q), p \to R$ and $q \to S$. (7 marks)

(ii) Show that the premises "A student in this class has not read the book and everyone in this class passed the first exam" imply the conclusion "some one who passed the first exam has not read the book".

(8 marks)

Turn over

III. (a) (i) If $f : A \to B$ and $g : B \to C$ are one-to-one onto functions, then prove that $g \circ f$ is also one-to-one.

(7 marks)

(ii) In a distributive lattice (L, α). Prove that if $a \wedge b = a \wedge c$ and $a \vee b = a \vee c$ implies $b = c$.

(8 marks)

Or

(b) (i) Show that the set Z of all integers is countable. (7 marks)

(ii) Let n be a positive integer and let D_n be the set of all positive divisors of n. Draw the Hasse diagram for D_{30}.

(8 marks)

IV. (a) (i) Let G be the set of all non-zero real numbers and let $a * b = \frac{ab}{2}$. Prove that (G, *) is an abelian group.

(7 marks)

(ii) State and prove Lagrange's theorem. (8 marks)

Or

(b) (i) Let H be a subgroup of a group G. Prove that every left coset aH of H has the same number of elements.

(7 marks)

(ii) State and prove Cayley's theorem. (8 marks)

V. (a) (i) Prove that a finite integral domain is a field. (8 marks)

(ii) Let R be a Euclidean ring, then prove that every element of R is either a unit in R or can be written as the product of a finite number of prime elements of R.

(7 marks)

Or

(b) (i) Show that if L is an algebraic extension of K and if K is an algebraic extension of F, then L is the algebraic extension of F.

(7 marks)

(ii) Prove that $x^3 - 9$ is inrreducible over F, the field of integer modulo 11.

(8 marks)

[4 × 15 = 60 marks]