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Name...

Reg. No.....................................

THIRD SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, DECEMBER 2008

PTCS/CS/IT 2K 301—ENGINEERING MATHEMATICS—III

Time : Three Hours

Maximum : 100 Marks

Answer **all** *questions.*

I. (a) Determine the subspace of $R^{(3)}$ spanned by $(i, 2, 1)$ $(2, 3, 0)$ and $(i, -1, -2)$.

(b) Find the product transformations $T_1 T_2$ and $T_2 T_1$ if $T_1(x, y, z) = (x + y, y + 2z, z + x)$ and $T_2(x, y, z) = (x + 3z, x - y, x + y + z)$ for all (x, y, z) in $R^{(3)}$.

(c) Determine the rank of the matrix :

$$\begin{pmatrix} 3 & -1 & 2 & 4 \\ 6 & 2 & -4 & -8 \\ 1 & 2 & 1 & 0 \\ 2 & 1 & -7 & -12 \end{pmatrix}.$$

(d) For the matrix $A = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$ find two non-singular matrices Q and Q^T such that Q^TAQ is a diagonal matrix.

(e) If $f(z) = u(x, y) + iv(x, y)$ is analytic and if u and v have continuous second partial derivatives in R, then prove that u and v satisfy Laplace's equation $\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = 0$.

(f) If $u = ln(x^2 + y^2)$ find the corresponding function V such that the analytic function $f(z) = u + iv$.

(g) Evaluate $\int_C \frac{z+1}{z^3 - 2z^2} dz$ where C is the circle $|z| = 1$.

(h) Find the region of convergence and the sum of the series :

$$\frac{1}{2}\left(\frac{z+1}{z-1}\right) + \frac{1}{2^2}\left(\frac{z+1}{z-1}\right)^2 + \frac{1}{2^3}\left(\frac{z+1}{z-1}\right)^3 + \ldots \text{ to } \infty.$$

(8 × 5 = 40 marks)

Turn over

II. (a) Determine whether the vectors (6, – 1, 6) (4, – 4, 1) (2, – 1, 1) are linearly independent or linearly dependent in R^3. Express (12, –8, 7) as a linear combination of the above vectors. (7 marks)

(b) Using Gram-Schmidt process, find an orthonormal basis for the subspace of $R^{(3)}$ spanned by the vectors (2, 4, – 4) $V_2 = (-3, 6, 0)$ and $V_3 = (7, 2, 1)$. (8 marks)

Or

(c) Find a basis for the range and its dimension of the linear transformation :

$$f(w, x, y, z) = (w + 2x + y + 3z, 2w - x + y + z, w + y + z).$$

(7 marks)

(d) If $T(x, y, z) = (x - y - z, 2x + z, x + 2y)$ denotes a linear transformation in R^3 find the inverse transformation of T. (8 marks)

III. (a) Classify the quadratic forms of :

(i) $f = 3x_1^2 + 3x_2^2 + 6x_3^2 - 2x_1 x_2 - 4x_1 x_3$.

(ii) $f = 2x_1 x_2 + 2x_1 x_3 + 2x_2 x_3 - x_1^2 - 3x_2^2 - 5x_3^2$. (7 marks)

(b) Find the characteristic values and the characteristic vectors of the matrix $\begin{pmatrix} 3 & -1 & 0 \\ -1 & 1 & -1 \\ 0 & -1 & 5 \end{pmatrix}$. (8 marks)

Or

(c) Determine the transformation that will reduce the quadratic form :

$$2x_1^2 + 13x_2^2 + 2x_3^2 + 4x_1 x_2 + 6x_2 x_3 + 2x_1 x_3$$

to a sum of squares. (7 marks)

(d) If $f(x) = x/(x + 4)$, compute $f(A)$ for the matrix $A = \begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & -2 \\ -2 & 2 & 3 \end{pmatrix}$. (8 marks)

IV. (a) **Show** that at the origin $f(z) = \begin{cases} \frac{x^3 - y^3}{x^2 + y^2} + i\frac{x^3 + y^3}{x^2 + y^2} & z \neq 0 \\ 0 & z = 0 \end{cases}$ satisfies Cauchy-Riemann equations, **but** does not have a derivative.

(7 marks)

(b) Derive the Cauchy-Riemann equation in polar co-ordinates γ and θ. (8 marks)

Or

(c) Find the bilinear transformation which maps the points $z = -1, 0, 1$ into the points $w = 0, i, 3i$.

(7 marks)

(d) Determine the image of the circles $|z| = 1$ and $|z| = 2$ under the transformation $w = \frac{z}{1-z}$.

(8 marks)

V. (a) Obtain Laurent series expansion for $f(z) = \frac{z}{(z-3)(z+2)}$ for $2 < |z| < 3$. (7 marks)

(b) Evaluate $\int_C \frac{e^z}{(z+1)(z+2)} dz$ if C is the circle $|z - 1| < 5$. (8 marks)

Or

(c) State and prove Cauchy's integral formula. (7 marks)

(d) Evaluate $\int_{-\infty}^{\infty} \frac{x^2}{(x^2 + a^2)(x^2 + b^2)} dx$ $a > 0, b > 0$ using Cauchy's residue theorem. (8 marks)

[4 × 15 = 60 marks]