

THIRD SEMESTER B.TECH. (ENGINEERING) DEG **EXAMINATION, DECEMBER 2008**

PTCS/CS/IT 2K 301—ENGINEERING MATHEMATICS—III

Time: Three Hours

Maximum: 100 Marks

Answer all questions.

- I. (a) Determine the subspace of $\mathbb{R}^{(3)}$ spanned by (i, 2, 1) (2, 3, 0) and (i, -1, -2).
 - (b) Find the product transformations T_1 T_2 and T_2 T_1 if T_1 (x, y, z) = (x + y, y + 2z, z + x) and T_2 (x, y, z) = (x + 3z, x y, x + y + z) for all (x, y, z) in $R^{(3)}$.
 - (c) Determine the rank of the matrix:

$$\begin{pmatrix} 3 & -1 & 2 & 4 \\ 6 & 2 & -4 & -8 \\ 1 & 2 & 1 & 0 \\ 2 & 1 & -7 & -12 \end{pmatrix}.$$

- (d) For the matrix $A = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$ find two non-singular matrices Q and Q^T such that Q^TAQ is a diagonal matrix.
- (e) If f(z) = u(x, y) + iv(x, y) is analytic and if u and v have continuous second partial derivatives in R, then prove that u and v satisfy Laplace's equation $\frac{\partial^2 \varphi}{\partial r^2} + \frac{\partial^2 \varphi}{\partial v^2} = 0$.
- (f) If $u = \ln (x^2 + y^2)$ find the corresponding function V such that the analytic function
- (g) Evaluate $\int \frac{z+1}{c^3-2z^2} dz$ where C is the circle |z|=1.
- (h) Find the region of convergence and the sum of the series:

$$\frac{1}{2} \left(\frac{z+1}{z-1} \right) + \frac{1}{2^2} \left(\frac{z+1}{z-1} \right)^2 + \frac{1}{2^3} \left(\frac{z+1}{z-1} \right)^3 + \dots \text{ to } \infty.$$

 $(8 \times 5 = 40 \text{ marks})$

Turn over

II. (a) Determine whether the vectors (6, -1, 6) (4, -4, 1) (2, -1, 1) are linearly independent or linearly dependent in \mathbb{R}^3 . Express (12, -8, 7) as a linear combination of the above vectors. (7 marks)

(b) Using Gram-Schmidt process, find an orthonormal basis for the subspace of R⁽³⁾ spanned by the vectors (2, 4, -4) $V_2 = (-3, 6, 0)$ and $V_3 = (7, 2, 1)$. (8 marks)

(c) Find a basis for the range and its dimension of the linear transformation:

$$f(w, x, y, z) = (w + 2x + y + 3z, 2w - x + y + z, w + y + z).$$

(7 marks)

- (d) If T(x, y, z) = (x y z, 2x + z, x + 2y) denotes a linear transformation in \mathbb{R}^3 find the inverse transformation of T.
- III. (a) Classify the quadratic forms of:

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(i)
$$f = 3x_1^2 + 3x_2^2 + 6x_3^2 - 2x_1 x_2 - 4x_1 x_3$$

(ii)
$$f = 2x_1x_2 + 2x_1x_3 + 2x_2x_3 - x_1^2 - 3x_2^2 - 5x_3^2$$
.

(7 marks)

(b) Find the characteristic values and the characteristic vectors of the matrix $\begin{pmatrix} 3 & -1 & 0 \\ -1 & 1 & -1 \\ 0 & -1 & 5 \end{pmatrix}$.

(8 marks)

Or

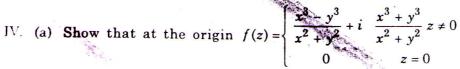
distribut and that the first (c) Determine the transformation that will reduce the quadratic form:

$$2x_1^2 + 13x_2^2 + 2x_3^2 + 4x_1x_2 + 6x_2x_3 + 2x_1x_3$$

to a sum of squares.

(7 marks)

(d) If f(x) = x/(x+4), compute f(A) for the matrix $A = \begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & -2 \\ -2 & 2 & 3 \end{pmatrix}$. (8 marks)



Satisfies Carent Riemann

equations, but does not have a derivative.

(7 marks)

- (b) Derive the Cauchy-Riemann equation in polar co-ordinates γ and θ .
- (8 marks)

Or

- (c) Find the bilinear transformation which maps the points z = -1, 0, 1 into the points w = 0, i, 3i.
- (d) Determine the image of the circles |z| = 1 and |z| = 2 under the transformation $w = \frac{z}{1-z}$.

(8 marks)

- V. (a) Obtain Laurent series expansion for $f(z) = \frac{z}{(z-3)(z+2)}$ for 2 < |z| < 3. (7 marks)
 - (b) Evaluate $\int_{C} \frac{e^{z}}{(z+1)(z+2)} dz \text{ if C is the circle } |z-1| < 5.$ (8 marks)

Or

(c) State and prove Cauchy's integral formula.

(7 marks)

(d) Evaluate $\int_{-\infty}^{\infty} \frac{x^2}{\left(x^2 + a^2\right)\left(x^2 + b^2\right)} dx \quad a > 0, b > 0 \text{ using Cauchy's residue theorem.}$ (8 marks)

 $[4 \times 15 = 60 \text{ marks}]$