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Name..............................

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THIRD SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION DECEMBER 2008

CH 2K 301—ENGINEERING MATHEMATICS—III

(Common to AI, CE, EE, IC, ME, EC, PE, PT, PTCE/PTEE/PTCH/PTME 2K 301)

Time : Three Hours Maximum : 100 Marks

I. (a) Test whether the following vectors are dependent or independent :—

(1, 1, 0, 0), (0, 0, 1, 1), (1, 0, 0, 4) and (0, 0, 0, 2).

(b) Find the linear transformation $T : V_3 \to V_3$ determined by the matrix

$$\begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ -1 & 3 & 4 \end{pmatrix}.$$

(c) Find the rank of the following matrix :—

$$A = \begin{bmatrix} 2 & 1 & 3 & 4 \\ -1 & 1 & 2 & 1 \\ 4 & 2 & 6 & 8 \\ -1 & 1 & -1 & 2 \end{bmatrix}.$$

(d) Find the eigenvalues of the matrix :

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}.$$

(e) Find the mean and variance for the Binomial distribution.

(f) The temperature during the month of June is normally distributed with mean 20° C and standard deviation 3.33 dg. Find the probability that the temperature is between 21.11° C and 26.66° C.

(g) The mean weight loss of $n = 16$ grinding balls after a certain length of time in mill slurry is 3.42 grams with S.D. and 0.68 gm. Construct the 99 % confidence interval for the true mean weight loss of such grinding balls under the same conditions.

(h) Two independent samples had the following values of the variables :—

Sample I : 10, 10, 9, 7, 11, 8, 9

Sample II : 11, 10, 12, 9, 8, 7, 11, 12

Do the estimates of the population variance differ significantly.

($8 \times 5 = 40$ marks)

Turn over

II. (a) (i) If x, y, z are linearly independent vectors, prove that $x+y, y+z, z+x$ are also linearly independent vectors.

(7 marks)

(ii) Find the matrix of the linear T on V_3 (R) defined by

$T(x, y, z) = (2y+z, x-4y, 3x)$

with the basis $a_1 = (1, 1, 1)$, a_2 (1, 1, 0), a_3 (1, 0, 0).

(8 marks)

Or

(b) (i) If w_1 and w_2 are subspaces of a vector space V such that $w_1 + w_2 = V$ and $W_1 \cap W_2 \{0\}$, prove that for each vector α in V there are unique vectors α_1 in W_1 α_2 in W_2 such that $\alpha = \alpha_1 + \alpha_2$.

(7 marks)

(ii) If T is a linear transformation on R^3 defined by $T(x, y, z) = (2x, 2x-5y, 2y+z)$, find T^{-1}.

(8 marks)

III. (a) (i) Find the eigenvalues and the eigenvectors of the matrix $A = \begin{bmatrix} 3 & -4 & 4 \\ 1 & -2 & 4 \\ 4 & -1 & 3 \end{bmatrix}$.

(8 marks)

(ii) If $f(x) = x^3 - 3x^2 - x + 9$, evaluate $f(A)$, for the matrix $A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & -1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$.

(7 marks)

Or

(b) Reduce the quadratic form $Q = 3x_1^2 + 2x_2^2 + 3x_3^2 - 2x_1 x_2 - 2x_2 x_3$ into sum of squares.

(15 marks)

IV. (a) (i) Find the mean and variance for the geometric distribution.

(8 marks)

(ii) Find the probability that atmost 5 defective will be found in a box of 200 fuses, if experiences shows that 2 % of such fuses are defective using Poisson distribution.

(7 marks)

Or

(b) (i) Find the mean for the Weibull distribution.

(7 marks)

(ii) A fair die is tossed 120 times. Use Chebyshev's inequality to find a lower bound for the probability of getting 80 to 120 sixes.

(8 marks)

V. (a) (i) The following random samples are measurements of the heat-producing capacity of specimens of coal from two mines.

Mian I : 8260, 8130, 8350, 8070, 8340

Mine II : 7950, 7890, 7900, 8140, 7920, 7840

Use 0.01 level of significance to test whether the difference between the means of these two samples is significant.

(8 marks)

(ii) Fit a Binomial distribution for the following data and test the goodness of fit :

x	0	1	2	3	4	5	6
f	5	18	28	12	7	6	4

(7 marks)

Or

(b) (i) Fit a Poisson distribution for the following data and test the goodness of fit :

x	0	1	2	3	4
f	123	59	14	3	1

(8 marks)

(ii) Two random samples drawn from two normal populations gave the following observations :—

Samples I : 20, 16, 26, 27, 23, 22, 18, 24

Samples II : 17, 23, 32, 25, 22, 24, 18, 31, 20.

(7 marks)

[4 × 15 = 60 marks]